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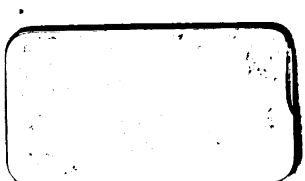

TEXT-BOOK OF ARITHMETIC,

FOR THE USE OF TEACHERS.

DESIGNED AS A

COMPANION AND SUPPLEMENT TO THE AUTHOR'S

MANUAL OF ARITHMETIC,

FOR THE USE OF PUPILS

IN NATIONAL AND OTHER ELEMENTARY SCHOOLS.

INCLUDING

SOLUTIONS OF THE MORE DIFFICULT EXERCISES IN THE MANUAL.

BY THE

REV. JOHN HUNTER,

VICE-PRINCIPAL OF THE NATIONAL SOCIETY'S TRAINING COLLEGE, BATTERSEA.

"That is to muche, and meete for no man,—to bee beleved in all thinges without shewinge of reason."—RECORDE'S *Grounde of Artes*, Pt. I. i.

"Nullâ re juvenum magis vegetatur ingenium, quàm numerorum arte discendâ."—TONSTALL, *De Arte Supput.*, Dedic.

LONDON:

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1847.

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A TEXT-BOOK of ENGLISH GRAMMAR, for Teachers.

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PREFACE.

THERE are two departments of elementary education which eminently conduce to the development of intellect—the sciences of Language and Number; and that they should be so taught as to constitute an intellectual discipline,—that they should not merely fill the memory with dogmatic Rules, but exercise the reason, has for some time been recognised by the advocates and expositors of sound education.

In the infancy of any science there is generally a tendency to extravagance and misdirection of Theory, and in its progress there frequently arise a disinclination to Theory, and a disposition to rest satisfied in gathering merely the results of demonstration. Such, at least, has been the case with the subject of the present volume. The mysticism of the Pythagoreans and the abstractions of the Boethian school carried Arithmetic beyond its legitimate province, and refused its application to the ordinary purposes of life; while, on the other hand, it must be admitted that the commercial school of arithmetical writers, which arose in this country in the sixteenth century, too much discouraged attention to Theoretical, and thereby retarded improvement in Practical Arithmetic.

During the long prevalence of those unreasoned methods of operation in which a course of arithmetical instruction consisted, there was one circumstance which tended to maintain some recognition of Arithmetic as an intellectual study,—the customary practice of constructing and collecting problems which had no immediate reference to business, and were designed chiefly to exercise the ingenuity of students.

The *Recreations* of Martin Clare, first published in 1739, in his *Youth's Introduction to Trade and Business*, and augmented in subsequent editions, formed a rich provision for the compilations of Vyse, Birks, Hay, &c.; and these, with a few additional Recreations by Welsh, Halbert, and others, were appropriated in greater or less quantity, in successive arithmetical productions, to nearly our own day. The uncouth language in which many of the problems were expressed, and especially the rhymes, in which it was often desperately attempted to marshal the terms of a question according to the harmonious *numbers* of the Muse, were suffered, indeed, to remain too long unimproved, and presented an unintellectual aspect to the eyes of any but those who could "by numbers judge a poet's song." But the elements of scientific amelioration were slowly developed by attempts to improve the solutions of such problems, by employing arithmetically demonstrative methods for many questions which had been committed to the determination of Position or of Algebra. And, eventually, the

substratum of improvement has been deprived of superficial deformities, till, like the inscription on the tower of Pharos when the coating of plaster had worn away, Arithmetic has assumed an aspect worthy of science, in the writings of such men as Thomson, De Morgan, Hind, and many others.

Perhaps one faulty condition of Arithmetic, as taught at present, is the unnecessary tediousness with which principles are sometimes inculcated, the small degree in which the demonstrative system is extended to the higher applicate forms of the science, and the limits of difficulty to which arithmetical problems are confined. The admirable Treatise by Professor Thomson of Glasgow is free from these objections. His uniform attention to the important object of making the rules of Arithmetic an intellectual discipline, and his admirable collection of Exercises, have left the Author of the present Text-Book no other apology for the publication of a new work on Arithmetic, than the warrantableness of an endeavour to carry forward the improvement of the science, by furnishing additional means of intellectual exercise to the more advanced student, and more suitably accommodating the circumstances of the ordinary school-boy.

The encouragement which the Pupil's Manual has met with, during the short period of its publication, has greatly exceeded the Author's expectation; but he hopes that that little work will be rendered still more acceptable and useful by the aids furnished in this Companion and Supplement. It will be seen

that the Author has not attached great importance to formal and general Rules, by which the intellect is liable to become, as it were, stereotyped rather than progressively improved. Concise directions may include much and impress little; but an abundant variety of Examples, such as the present Work contains, will, it is hoped, conduce in the most effectual manner to a perception of general principles, and to expertness in the application of them.

The Author will only further state that, having employed his own best efforts to secure the important objects of expedient arrangement, clear exposition, and concise and accurate solution, he is anxious to avail himself of any hints for the improvement of this Work, which the skill and experience of other Teachers may suggest.

Nat. Soc. Training College, Battersea,
August, 1847.

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ARITHMETIC.

ARITHMETIC teaches the nature of numbers, and how to increase, diminish, and compare them.

Its processes are usually conducted by means of convenient marks that stand as signs for numbers.

Before, however, the pupil's attention is directed to the *figures* commonly employed for the notation of numbers, he may be taught the nature of the operations termed Addition, Subtraction, Multiplication, and Division, by reference to a row of ten units, or counters, chalked on a black board. We proceed to indicate a form of instruction for this purpose, recommending the teacher to refer with a pointer to the row of units, while he gives the lessons, and to encourage the pupils to name the sum, remainder, &c., as he goes along. Promiscuous examination should follow each lesson, first with the aid of the counters, and afterwards without it.

INTRODUCTORY EXERCISES.

ROW OF UNITS: | | | | | | | | | |

A single thing is referred to by the number *One*.

ADDITION.— *One* together with another *one* makes the number *two*; *two* together with *one* makes *three*; *three* together with *one* makes *four*; and so on, until *ten* is the sum attained.

One together with *two* makes *three*; *two* together with *two* makes *four*; *three* together with *two* makes *five*; &c.

One together with *three* makes *four*; *two* with *three* makes *five*; *three* with *three* makes *six*; &c.

Examination.—*Four* apples and *one* apple make how many? Add together *three* and *two*. Add *four* and *one* and *three*. What is the sum of *two* pennies and

five pennies? Put into one sum *three* and *three* and *four*.

SUBTRACTION.—*Ten* when *one* is taken from it becomes *nine*; *nine* lessened by *one* is *eight*; *eight* lessened by *one* is *seven*; &c.

Ten lessened by *two* becomes *eight*; *nine* lessened by *two* is *seven*; &c.

Examination.—From *nine* take *five*, what is the number left? From *seven* take *two*, what remains? *Four* and what make *ten*? Subtract *nothing* from *eight*; *three* from *eight*. What three numbers make up *ten*?

MULTIPLICATION.—*One* taken *one* time, or *once*, is *one*; *one* taken *two* times, or *twice*, is *two*; *one* taken *three* times is *three*; &c.

Two taken *once* is *two*; *two* taken *twice* is *four*; *two* taken *thrice* is *six*; &c.

Examination.—*One* and *one* and *one* are how many times *one*? How many are *three* times *two*? *Four* times *one* and *three* times *one*? Multiply *five* by *two*; *two* by *five*. Take *two* from *three* times *two*. Take *four* from *five* times *one*. Take *four* from *five* times *two*. When I say, Add *one* to *three* times *two*, how many operations are proposed? Their names? When I say, From *four* times *one* take *two*, what two operations are proposed? Subtract *ten* from *five* times *two*.

DIVISION.—*One* is contained in the number *ten*, *ten* times; *one* is contained in *nine*, *nine* times; *one* is contained in *eight*, *eight* times; &c.

Two is contained in *ten*, *five* times; *two* is contained in *nine*, *four* times (with *one* over); &c.

Examination.—How often is *five* contained in *ten*? in *seven*? in *five*? in *three*? Divide *eight* by *four*; *five* by *one*; *ten* by *six*. If *six* is divided into *two*

equal parts, what is the value of each part? What is the *third* part of *nine*? the *tenth* part of *ten*? the *half* of *eight*? the *fifth* of *ten*? How many *tenth* parts of *ten* make the number *four*? How many *third* parts of *six* make the number *eight*? How many *third* parts of *nine* make the number *nine*? Add *four* to the *half* of *four*. Subtract *one* from the *fifth* of *five*. Multiply *three* by the *fourth* part of *eight*; by the *ninth* of *nine*.

NOTATION AND NUMERATION.

The figures 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, are convenient signs, used instead of the words *nothing*, *one*, *two*, *three*, *four*, *five*, *six*, *seven*, *eight*, *nine*; and by means of these figures any number may be expressed.

For *Definitions*, see Manual, p. 1.

A single figure, or the right-hand figure of a number, represents so many *tenth-parts* of *ten*, or *units*, or *ones*; the second figure from the right signifies so many *tens*; the third so many *tens* of *ten*; the fourth so many *tens* of *tens* of *ten*. Thus—

7 is seven *units*, or *tenth-parts* of *ten*;
70 is seven *tens*, and no *units*;
63 is six *tens*, and three *units*;
428 is four *tens* of *ten*, two *tens*, and eight *units*;
3505 is three *tens* of *tens* of *ten*, five *tens* of *ten*, and five *units*.

All whole numbers, therefore, are represented by figures which express so many *tenth-parts* of *ten*, or so many *whole tens*, or so many combined *tens* and *tenths* of *ten*.

In naming the value of a number, of more than one figure, the usual method is to read it from left to right.

The name *tenth-parts* of *ten*, or *units*, or *ones*, is generally not mentioned. Thus, 7 is simply called *seven*.

The figures representing simple *tens* are named in

the following manner: 10 ten, 20 twenty, 30 thirty, 40 forty, 50 fifty, 60 sixty, 70 seventy, 80 eighty, 90 ninety.

The figures representing *tens* of *ten* are usually called *hundreds*. Thus: 100 one hundred, 200 two hundred, &c.

The figures representing *tens* of *tens* of *ten* (or, *tens* of *a hundred*) are usually called *thousands*. Thus: 1000 one thousand, 2000 two thousand, &c.

The numbers between *ten* and *twenty* are represented and named thus: 11 eleven, 12 twelve, 13 thirteen, 14 fourteen, 15 fifteen, 16 sixteen, 17 seventeen, 18 eighteen, 19 nineteen.

The teacher should now illustrate the foregoing principles and usages by promiscuous examples; and should ascertain by examination that the pupil thoroughly understands the scheme of notation so far as thousands; it is, for the present, inexpedient to exceed that limit.

Promiscuous Examples.—The first number consists of 3 tens, and 5 tenths of ten; and it is named *thirty-five*.

The second number consists of 3 tens of ten, 5 simple tens, and no tenths of ten; and it is named *three hundred and fifty*.

&c. &c.

35
350
472
601
717
2748
5060
9911

It may now be more distinctly stated:—

That every figure becomes *ten times* as great in value, by being shifted to the place next it on the left:—Compare 32 and 23, for instance.

That, *therefore*, nine significant figures, together with *zero*, or the cipher 0, are sufficient to express all values.

That the cipher, which has by itself no value, is used to mark the place where any value is wanting, in order to secure a proper local value for the figure or figures on its left. Compare, for instance, 30 with 3.

That values ascending from *zero* to *nine* (whether they mean so many units, or so many tens, &c.) are represented by *different figures*; and that *ten*, the next

higher value, is represented by a *difference of position.*

When the Numeration and Notation Exercises in the Manual have been satisfactorily performed, the pupil should be prepared (by such instruction as is given in next section) to work the first forty Exercises in Simple Addition; and after performing these, he may be taught (by reference to the Table at the foot of page 1 of the Manual) the numeration of values higher than thousands.

SIMPLE ADDITION.

Exemplification :—Suppose I have 3 shillings, and you give me 5 shillings more, I shall then have 8 shillings; if I get 4 shillings besides, I shall have 12 shillings. Here I have increased the number 3 by combining it with 5, and further increased it by combining it also with 4.

For *Definition* of Addition, see Manual.

Before slate-exercises in Addition have been entered upon, and occasionally afterwards, the pupil should be practised in the mental addition of simple units of which the final amount does not exceed 100.

Addition is often signified by the mark +, which is called the sign of addition, and named *plus* (a Latin word which denotes *more*). We may, accordingly, represent the addition of 3, 5, and 4, by writing 3 + 5 + 4 equal to 12; or, employing another mark of operation, called the sign of equality, 3 + 5 + 4 = 12, which is read 3 *plus* 5 *plus* 4 *equal to* 12.

Let the pupil be required to represent by the above signs of operation, the addition of two or more of the first twelve Exercises in the Manual.

When several large numbers are proposed for addition, they are usually arranged in successive lines, all the units forming one column, the tens another column, &c.

For example, to find what sum the following expression is equal to; 38 + 59 + 47 + 26 + 75:—

Let the given numbers be arranged as in the margin; and let the columns of units and tens be added separately. It is usual to commence at the bottom, and to add upwards; although the sum might be found by any order of addition. Add, then, first the units; 5 and 6 make 11, and 7 more make 18, and 9 more make 27, and 8 more make 35 units, which is the sum of the first or right hand column. Now 35 units consist of 3 tens of units and 5 simple units; therefore, write 5 under the units column, and combine the three tens with the column of tens; thus, 3 and 7 are 10, and 2 are 12, and 4 are 16, and 5 are 21, and 3 are 24 tens; that is, 2 tens of ten and 4 simple tens, or 2 hundreds and 4 tens; therefore the whole sum is 245.

Tens.	Units.
3	8
5	9
4	7
2	6
7	5
2	4 5

For illustration's sake, the above example may be exhibited also in the following manner:—

38	=	3 tens	+	8 units				
59	=	5 tens	+	9 units				
47	=	4 tens	+	7 units				
26	=	2 tens	+	6 units				
75	=	7 tens	+	5 units				
		21 tens	+	35 units				
But 21 tens	=	2 hundreds	+	1 ten				
35 units	=			3 tens	+	5 units		
Ans. 245 units	=	2 hundreds	+	4 tens	+	5 units		

The number found by Addition is called the *sum*, *amount*, or *aggregate*; and the numbers added are called the parts of the aggregate, or the *aggregate parts*.

The *Reason* of the Rule of Addition is simply the axiom, that any whole is equal to the sum of its aggregate parts.*

* When a number is also equal to the sum of all its *aliquot* parts, it is called a *Perfect Number*. Thus the aliquot parts of 6 are 3, 2, and 1; and therefore 6 is called a Perfect Number. So also 28 is a Perfect Number, being equal to 14 + 7 + 4 + 2 + 1.

As a good method of *proving* the Pupil's accuracy in the ordinary process, he may be required to begin at the top of the left-hand column, and add downwards. Let the carrying figures be reserved in a second line, as in the margin; and, finally, add together the two lines thus produced. The ordinary method of proving Addition, by reserving the head number for combination with the sum of the remaining numbers, does not secure the Pupil against the recurrence of mistakes.

38
59
47
26
75
—
245
—
15
23
—
245

The teacher should not be content with ascertaining that the pupils have wrought the Exercises in Addition, but should occasionally call upon them, after they have silently performed some particular Exercise, to go over the steps of the calculation audibly, and to show that they understand the numeration of the figures. This suggestion applies to all the elementary rules.

SIMPLE SUBTRACTION.

Exemplification:—If I have 8 nuts, and give away 2 of them, I shall have 6 remaining.

If I have 10 apples, and you have 6 apples, I have 4 apples more than you. I thus compare my number with yours; and, by supposing 6 of my apples to be taken away, I observe that I should have 4 left; whereas, if 6 of your apples were taken away, you should have none left. I have, therefore, 4 more than you, or 4 is the difference of our numbers.

For *Definition* of Subtraction, see Manual.

Before slate-exercises in this Rule, let the pupil be practised in mental subtractions, and be made acquainted with the terms *difference, excess, remainder*, &c., by varied forms of interrogation.

Subtraction is often indicated by the mark −, which is named *minus* (a Latin word which signifies *less*). We may, accordingly, represent the subtraction of 9 from 13 by writing 13 − 9 = 4, which is read 13 *minus* 9 equal to 4.

When two large numbers are given to find their difference, it is usual to place the greater over the less, so that units may be subtracted from units, tens from tens, &c., separately.

For example, let it be required to find what single number is equal to 8956 — 452.

Placing the less number under the greater, as in the margin, we say 2 units from 6 units leaves four units; 5 tens from 5 tens leaves 0 tens; 4 hundreds from 9 hundreds leaves 5 hundreds; 0 thousands from 8 thousands leaves 8 thousands. Therefore the entire remainder is 8504.

Thous.	Hund.	Tens	Units
8	9	5	6
	4	5	2
8	5	0	4

When the pupil has performed the first five exercises in the Manual, his attention may be called to an Example such as the following:—

Subtract 379 from 862.

Here, in proceeding to subtract 9 units, we perceive that the upper units do not admit of such subtraction. However, if we increase the upper number by 10 *units*, and also increase the lower number by 1 *ten*, the difference of the two enlarged quantities must be the same as that of the given quantities.* This makes

862	
379	
483	

the upper number consist of 8 hund., 6 tens, 12 units,
and the lower number . . 3 hund., 8 tens, 9 units.

But, though the *units* are now practicable for subtraction, the *tens* are not. Let us, therefore, add 10 *tens* to the upper, and 1 *hundred* to the lower quantity, and this will make the

upper number consist of 8 hund., 16 tens, 12 units,
and the lower number . 4 hund., 8 tens, 9 units.

The difference, therefore, is 4 hund., 8 tens, 3 units.

* Let this uniformity be made distinctly evident to the pupil by a few examples, such as—Harry has 7 apples and Tom has 12; how many has Tom more than Harry? Give each of them 4 apples more; then Harry's number is 11, and Tom's is 16; how many has Tom more than Harry?

Accordingly, whenever the value of a figure in the upper line is greater than that of the figure to be subtracted, conceive 10 to be added to the upper, and 1 to be added to the next lower.

The number to be lessened is called the *minuend;* and the number to be subtracted, the *subtrahend.*

The *reason* of the Rule of Subtraction is obviously implied in that of the Rule of Addition.

To *prove* the operation, add the remainder to the subtrahend; the sum should be equal to the minuend. The truth of this is manifest in the fact, that a portion taken away from a quantity, and the portion remaining, must, when recombined, make up that quantity.

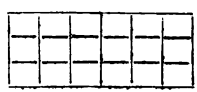
The above example may be stated by means of the signs of operation; thus, $862 - 379 = 483$; and the pupil may very easily, and with advantage, be taught to vary the relations of the numbers without destroying the equation; thus $862 = 483 + 379$, which represents the method of proof; also $862 - 483 = 379$. These variations should be introduced to the pupil's notice, and exemplified by him, when he arrives at Ex. 29th.

It will be observed that Exercises 39 to 43 have the minuend placed under the subtrahend. As such an arrangement is often occasioned in the solution of problems, it may be expedient for the pupil to practise a little subtraction in that way.

SIMPLE MULTIPLICATION.

Exemplification:—If I should buy 6 copy-books at 3 pence each, I should pay 6 times 3 pence, or 18 pence, altogether. Or, if I should buy 3 copy-books at 6 pence each, I should pay 3 times 6 pence, or 18 pence, altogether.

It appears, then, that 6 times 3 equals 3 times 6; and we can always interchange the numbers in this manner—as will be evident from the annexed diagram; for the 18 squares, of which it is composed, are made up of 3 rows of 6 squares each (*i. e.* 3 times 6 squares), or 6 rows of 3 squares each (*i. e.* 6 times 3 squares).



For *Definition* of Multiplication, see Manual.

Make the pupil understand, from the above diagram, and by one or two numeral examples, that Multiplication is an expeditious method of producing the sum of several equal numbers; and that it requires the elementary products, which compose the Multiplication Table, to be carefully learned and readily recollected.

The sign of Multiplication is $\times$; thus, $5 \times 4 = 20$, signifies that 5 *multiplied by* 4 *equals* 20.

The mode of extending the application of the table of elementary products is now to be exemplified.

Suppose we are required to multiply 3475 by 3.

This, of course, proposes to find the sum of 3475 repeated 3 times, and might be answered by addition; but the method by multiplication is more convenient, and is thus performed:—

Placing the number of times under the number to be repeated, we say, 3 times 5 units are 15 units, that is, 1 ten and 5 units; set down 5 units, and carry 1 ten; 3 times 7 tens are 21 tens, and 1 ten reserved makes 22 tens; set down 2 tens, and carry 2 tens of tens, or hundreds; 3 times 4 hundreds are 12 hundreds, and 2 reserved hundreds make 14 hundreds; set down 4 hundreds, and carry 1 ten of hundreds, or thousand; 3 times 3 thousands are 9 thousands, and 1 thousand carried makes 10 thousands; set down 10 thousands entire.	3475 3 10425

The *reason* of this process is the same as for addition.

The sum 10425, when obtained by the process called Multiplication, is said to be the *product* of 3475; also, 3475 and 3 are called the *factors* (*i. e. producing* numbers), the upper factor being specially termed the *multiplicand* (*i. e.* the number *to be multiplied*), and the lower factor the *multiplier*.

In every *arithmetical* multiplication, the multiplier denotes simply *so many times* the multiplicand. Thus, to give 6 *shillings* to each of 3 *men*, we require 3 *times* 6 shillings, or 6 shill. $\times$ 3; but we cannot say 3 *men times* 6 shillings; nor is it proper to

write such an expression as 6 shill. × 3 men, unless merely by accommodation, to indicate whence the multiplier 3 is derived. For the purpose of arithmetical multiplication, the quantity, 3 men, has the notion of *men* abstracted or drawn away from it, leaving what is therefore called an *abstract* number. The notion of 3, when applied to and united with the notion of *men*, becomes 3 men, which is therefore called an *applicate*, or *concrete* (i. e. *compounded*) number.*

Hence, it may be stated, that no number can be used as a multiplier except as an *abstract* number,—that the multiplicand *may* be a *concrete* number,—and that the product *must* be originally of the same name as the multiplicand.

When the pupil has performed the first twenty Exercises, and has proved his work, in some instances at least, by Addition, his attention may be directed to the following Examples:—

Examp. Multiply 478 by 10.

First, let the product be constructed by the usual successive steps; and then, by examination of the figures which are successively carried, it will appear, that to multiply any number by 10 requires only to annex 0 to the multiplicand. It is obvious, indeed, from the principle of our decimal notation, that annexing 0 at the right of the multiplicand, makes *each figure* of it ten

478
10
4780

* *Geometrical* multiplication, or the generation of surfaces and solids, is different from *Arithmetical*. Sir Isaac Newton says, "Multiplication is made not only by *abstract* numbers, but also by *concrete* quantities, as by lines, surfaces, local motion, weights, &c., as far as these may be conceived to express or involve the same ratios to some other known quantity of the same kind, esteemed as unity, as numbers do among themselves. Custom has obtained, that the *genesis*, or description, of a surface, by a line moving at right angles upon another line, shall be called the multiplication of those two lines. For though a line, however multiplied, cannot become a surface, and, consequently, this generation of a surface by lines is very different from multiplication, yet they agree in this, that the number of unities in either line, multiplied by the number of unities in the other, produces an abstracted number of unities in the surface comprehended under those lines, if the superficial unity be defined, as it used to be, viz. a square whose sides are linear unities."—*Arithmetica Universalis*, translated by *Raphson*, p. 4. See also *Colenso's Arithmetic*, chap. i. art. 16.

times as great in value, and consequently gives an *entire product* ten times as great as the multiplicand.

Examp. Multiply 478 by 16.

This multiplier is beyond the limits of the Multiplication Table, but may be accommodated to the Table in various ways. For 16 times a number is the amount of 6 times + 10 times that number, or of 11 times + 5 times that number, or of 9 times + 7 times, &c. We shall exemplify the first and third of these considerations.

478	478		478	478	
6	10		9	7	
2868	4780	= 10 times	4302	3346	= 7 times
	2868	= 6 times		4302	= 9 times
	7648	= 16 times	=	7648	

Again, 16 times a number is 8 times the *double* of that number; therefore, we may first find the amount of 2 times the number, and then multiply that amount by 8. Or, 16 times a number requires a product 4 times as great as 4 times that number, because $4 \times 4 = 16$; therefore, we may first find the amount of 4 times the number, and then multiply that amount by 4. We shall exemplify both these modes of consideration.

478		478	
2		4	
956	= 2 times	1912	= 4 times
8		4	
7648	= 16 times =	7648	

Examp. Multiply 5274 by 30.

As 30 times is ten times *thrice*, we may first take 3 times the multiplicand, which gives 15822, and then take 10 times that product, which requires simply to annex 0. By examination of the process it is seen

5274
30
158220

that to multiply by an exact number of tens, we may multiply by the tens figure, as if it denoted units, and then give the product a tenfold value by annexing 0 to the right.

In like manner, it may be shown that to multiply by an exact number of hundreds, thousands, &c., we may first multiply by the figure denoting the hundreds, &c. as if it denoted units, and then annex to the product two ciphers for hundreds, three for thousands, &c. If ciphers occur on the right of the multiplicand, they may be considered as transferred to the right of the multiplier. Thus 4600×30 is $46 \times 100 \times 30$, or 46×3000.

The form usually adopted for multiplication, when ciphers are on the right of the multiplier, is exhibited in the following examples:—

2573	2573	2573000
40	4000	400
102920	10292000	1029200000

The pupil is now required to perform the second set of Exercises, Nos. 21 to 36; and for most of these he should employ two modes of solution, such as we shall here exemplify by working the 29th Exercise:—

1916377	1916377
56	8
11498262 = 6 times	15331016 = 8 times
95818850 = 50 times	7
107317112 = 56 times	107317112 = 56 times

The latter method, when the multiplier is a product of two factors, neither exceeding 12, will serve as a means of *proof*, but is not to be regarded as being always the preferable mode of solution for Simple Multiplication. It has a superior expediency in Compound Multiplication.

The third set of Exercises does not conveniently

admit of solution by the latter of the above methods, as no one of the multipliers is the product of two factors occurring in the Table. The pupil must now, therefore, give chief attention to the former method, which is general in its application, and is usually simplified by the omission of those right-hand ciphers that are uniformly occasioned by the tens, hundreds, &c. of the multiplier. We subjoin three examples, excluding the ciphers from the last, and connecting it with a mode of proof which we shall presently explain:—

46957	36859	5864...	5
468	64005	2054...	2
375656	184295	23456	10
2817420	147436000	29320	
18782800	2211540000	11728	
21975876	2359160295	12044656...	1

In these examples, we find the entire product by the addition of three partial products. By the first operation we add 8 times + 60 times + 400 times the multiplicand; by the second we add 5 times + 4000 times + 60000 times the multiplicand; by the third we add 4 times + 50 times + 2000 times the multiplicand.

A method of *proof* called *casting out the nines*, is often applied to Multiplication. Thus, for the third of the above examples, add the figures of the multiplicand, as if each represented units; drop *nine* as often as it arises in the summing, and place the result a little to the right:—According to this process, we say 5 and 8 gives 4 (*i. e.* 13 dropping 9), and 6 gives 1, and 4 gives 5. Place 5 on the right. Apply the same process to the multiplier and the product; the results will be 2 and 1. Lastly, multiply together the two uppermost results, which gives ten; and, as casting out the 9 from this leaves 1, which has already resulted from

the main product, the exercise is presumed to have been correctly performed.

Casting out the *nines* is only a *presumptive* test of accuracy, as errors may have been committed which it cannot detect. It is, however, generally to be depended on, and is recommended by its simplicity.*

SIMPLE DIVISION.

Exemplification:—Suppose that with 18 shillings I have bought some articles which cost 3 shillings a-piece; the number of articles is 6; because 3 shillings are contained in 18 shillings 6 times.

Or, suppose I have 18 shillings to divide equally among 6 persons, and I am to consider what each person should receive; I ask what number of shillings is contained 6 times in 18 shillings, or what is the 6th part of 18 shillings; and I find the answer to be 3 shillings.

There are therefore two ways of proposing a question in Division. See *Definition* (Manual).

As Multiplication is a short method of doing a par-

* Any number, used as a *Divisor* of the multiplicand, multiplier, and product, would furnish a presumptive test; but as the pupil is not yet considered capable of performing Division, the number 9 is chosen as one which leaves the same remainder, after dividing a number, as it would leave after dividing the sum of the digits of that number. The number 3 also has this peculiarity.

200 ÷ 9 leaves 2
80 ÷ 9 leaves 8
4 ÷ 9 leaves 4
284 ÷ 9 leaves 5

We subjoin a demonstrative illustration of the applicability of 7 as a testing divisor. The process will be seen to involve a general demonstration.

Because $284 = 4 \quad + \text{a multiple of } 7;$
12
Therefore $\overline{3408} = 4 \times 12 + \ldots$ ditto;
But $12 = 5 \quad + \ldots$ ditto;
Therefore $3408 = 4 \times 5 + \ldots$ ditto;
$= 6 + \ldots$ ditto.

ticular species of Addition, so Division is a short method of doing a particular species of Subtraction.

The first example in the margin supposes a person, originally possessing *no* pence, to receive three several sums of 8 pence; the second supposes a person originally possessing 24 pence to give away 8 pence as many times as his money permits. Such examples may be employed to illustrate the relation of Multiplication to Division, and their respective relations to Addition and Subtraction. By the first process it is found that 3 times 8 make 24; by the second, that 24 contains 8 exactly 3 times.

0	24
8	8 *one* time
8	16
8	8 *two* times
16	8
8	8 *three* times
24	0

It is evident, then, that as Addition is the reverse of Subtraction, and a knowledge of the former is our guide in performing the latter, the same relation obtains between Multiplication and Division. We know that 6 subtracted from 9 leaves 3, because 6 added to 3 make 9; so also we know that 18 divided by 6 gives 3, because 6 multiplied by 3 gives 18.

As Multiplication proposes two factors, and requires their product, Division proposes the product and either factor, to find the other factor; thus 30 divided by 6 gives 5, or divided by 5 gives 6. For illustration of this interchange of divisor and quotient, reference may be made to the diagram given under Multiplication.

It is, therefore, by having learned the Multiplication Table, that we are enabled to perform Division. But though a thorough acquaintance with that Table implies a ready power of naming the factors which make any of its products, the pupil, when entering upon the study of Division, should be practised by such questions as these:—5 times what number make 30? How often is 6 contained in 48? How often is 7 contained in 34? Is 19 exactly divisible by 8? Name a number exactly divisible by 3; &c.

The name *dividend* is employed to denote the num-

ber to be divided; the *divisor* is the number by which we divide; and the *quotient* is the number of times which the dividend contains the divisor.

The sign of Division is $\div$; thus, $56 \div 8 = 7$. Frequently, the upper and under dots of the sign are replaced by the dividend and divisor, respectively; so that we may write $\frac{56}{8} = 7$. Both expressions signify that 56 *divided by* 8 *equals* 7.

As in Multiplication the proper multiplier is always *abstract*, so in Division the proper quotient is always *abstract*. Also, the dividend and divisor, like the multiplicand and product, are always of one name. Therefore, in such an expression as 12 yards $\div$ 3 = 4 yards, we should account 3 as being properly the quotient, *i. e.* the number which indicates *how often* the required quantity shall be contained in 12 yards.

The division of large numbers is conveniently performed by using a succession of partial dividends. We shall exemplify first with divisors not exceeding 12.

Divide 9774 by 3; and 23038 by 5.

Write the divisor on the left of the dividend, and draw lines of separation as in the margin. Then 3 in 9 thousand goes 3 thousand times; 3 in 7 hundred goes 2 hundred times, leaving 1 hundred over, which, as being equal to 10 tens, makes the next partial dividend 17 tens; 3 in 17 tens goes 5 tens of times, leaving 2 tens over, or 20 units, making the next partial dividend 24 units; 3 in 24 units goes 8 times. We find, then, that 3 is contained in 9774 exactly 3258 times.

```
3)9774
 ----
 3258
```

In the second example the first partial dividend to be taken is 23 thousand, as 2 by itself is not a sufficient number of tens of thousands to yield a quotient figure of that higher denomination. At the end of the division we find 3 units remaining over to be divided by 5; but as the 5th part of 3 is not a whole number, that

```
5)23038
 ------
 4607 3/5
```

part of the quotient, represented under the form of division, is called a fraction (*i. e.* part of a *broken* quantity), and is read 8 *fifths*. Accordingly, 5 is contained in the proposed dividend 4607 times and 3 fifth parts of a time.

When the pupil has wrought the first twenty exercises, and has, occasionally at least, proved his work, by multiplying the quotient by the divisor, and adding to the product whatever remainder may have been left undivided, he may proceed to the consideration of Composite Divisors, *i. e.* Divisors composed of two factors each greater than unity.

When a divisor is a composite number, we may divide successively by its factors. When, for example, 27 is the divisor, we may divide first by 3 and then by 9.

If we mark 27 counters, it will be evident that, as the 3rd part of them is 9, and the 9th part of 9 is 1, therefore, the 27th part of them (viz. 1) is equal to the 9th part of the 3rd part of them. Or thus: When we have found how many divisions of 3 the dividend contains, we can argue that the divisions of 27 are one 9th part as many as the divisions of 3.

Let it be required to find what is the *twenty-eighth* part of 10220; also, how many *twenty-sevens* there are in 68769.

7)10220	3)68769
4)1460 = the 7th of 10220	9)22923 *threes* in 68769
365 = the 4th of the 7th	2547 *twenty-sevens*

Suppose, next, that we are required to find how many *thirty-sixes* are in 13894.

4)13894 *units*	2 units + 8 fours = 34 units, remainder.
9)3473 *fours* + 2 *units*	
385 *thirty-sixes* + 8 *fours*	*Ans.* 385$\frac{8}{9}$ *thirty-sixes.*

The above example will evince the reason of the ordinary Rule for collecting the entire remainder from partial remainders; the Rule being, to multiply the first divisor by the second remainder, and add to the product the first remainder. The extension of that Rule to suit an example like the following will be easily understood.

Find the quotient of 18212 ÷ 216.

The divisor is equal to 4 × 6 × 9; therefore,

4)18212 *units*

6)4553 *fours*

9)758 *twenty-fours* + 5 *fours*

84 *two hund. and sixteens* + 2 *twenty-fours.*

The full remainder consists of 5 fours + 2 twenty-fours = 20 + 48 = 68.

In the following example it will be seen that, to divide by 10, we need only to consider the dividend, without its units figure, as the quotient, and the units figure as the remainder. Similar simplicity pertains to the divisors 100, 1000, &c.

How often is 80 contained in 47547?

10)47547

8)4754 *tens* + 7 *units*

594 *eighties* + 2 *tens*

7 units + 2 tens = 27 units, remainder.

Ans. $594\frac{27}{80}$ times.

To save figures, the above example may be wrought as in the margin, by cutting off the units figures of divisor and dividend, and dividing at once by 8.

8,0)4754,7

$594\frac{27}{80}$

These examples and explanations will prepare the pupil for the solution of Exercises 21 to 36; after which he will enter upon

the most general method of Division, commonly called Long Division.

The operations we have already exemplified are those of Short Division. We shall illustrate the method of Long Division by an example which can be wrought by Short Division.

Required the quotient of 28760 ÷ 8.

Here we see first, that 2 tens of thousands, divided by 8, gives no tens of thousands for the quotient. Accordingly, we must take 28 thousand as the first partial dividend suiting the divisor; and we find that the highest number of thousands which division by 8 will yield is 3; therefore, as 3 thousand times 8 is 24000, there remains yet, of the dividend, 4760; of this we take 47 hundred as the second partial dividend suiting the division; and the highest number of hundreds which it gives when divided by 8 is 5; then as 5 hundred times 8 is 4000, the remainder of the dividend is 760; of this we take 76 tens, yielding for the quotient 9 tens; and so forth, till the entire dividend is exhausted, and the entire quotient is found to be 3595.

8)28760		(3000
	24000	
	4760	500
	4000	
	760	90
	720	
	40	5
	40	
	—	3595

It is manifest, however, that such a mode of division is necessary only when the divisor employed is a number too great to allow the successive multiplications and subtractions to be performed, with ease, mentally.

It is also manifest, that the places which uniformly belong to ciphers may be left blank; and hence the subjoined specimens exhibit the most convenient methods of performing long division. The second form is preferred by the author as assigning to the quotient a position more natural, and more conducive to accuracy in bringing down the successive figures of the dividend.

Divide 1823117 by 479.

479)1823117(3806$\frac{43}{479}$

1437

3861

3832

2917

2874

43

479)1823117

3806$\frac{43}{479}$

1437

3861

3832

2917

&c.

ARTIFICES FOR ABBREVIATING SIMPLE MULTIPLICATION.

In many cases of Simple Multiplication the factors are such that we can easily obtain the entire product, without finding so many partial products as there are digits in the multiplier.

1. The multipliers 5, 25, and 125, are, respectively, the half of 10, the fourth of 100, and the eighth of 1000; therefore, conceive one cipher to be annexed to the multiplicand, and take half, for 5 times; conceive two ciphers to be annexed, and take a fourth, for 25 times, &c.*

2. The multipliers 99, 999, &c., are respectively, one less than 100, 1000, &c.; therefore, annex two, three, &c. ciphers to the given multiplicand, and then subtract that multiplicand.

Examp. Multiply 483 by 25, and by 99.

4)483

Ans. 12075

48300 = 100 times

483 = 1 time

Ans. 47817 = 99 times.

* Division may also be abbreviated when 5, 25, or 125, is the divisor; for we may divide twice, four times, &c. the given dividend by 10, 100, &c. respectively.

3. A multiplier can sometimes be resolved into aggregate parts, whose significant figures are so conveniently proportioned to each other, that the product by one aggregate part whose significant portion exceeds 12, may be readily obtained from the product by another whose significant portion does not exceed 12.

Examp. Multiply 5974 by 273, and by 756.

5974	5974
273	756
17922	41818
161298	334544
1630902	4516344

In the former example, we first compute for 3 times, and secondly, for 27 times, which is 3 times × 9; the product from 3, therefore, is multiplied by 9, to obtain 27 times the given multiplicand.

In the latter example, we first compute for 7 times, and secondly, for 56 times, which is 7 times × 8; the product from 7, therefore, is multiplied by 8. It is always allowable to take the partial products in any order of succession, provided we are careful that the units of each product stand under the right-hand figure of the multiplier from which it arises, or stand as many places from its multiplier as the other products from their multipliers.

Examp. Multiply 9612 by 987, and 86957 by 32849.

987	86957
9612	32849
11844	695656
94752	2782624
9487044	4173936
	2856450493

In the former example, we first find the product by

12, and then take 8 times that product, because 8 times 12 make 96.

In the latter example, we first compute for 8 times; then for 32 times, viz. 8 times $\times$ 4; then for 48 times, viz. 8 times $\times$ 6; but in adding the partial products, we include *once* the multiplicand, to complete 49 times.

Examp. Multiply 49378 by 7854, and 5973 by 4635.

49378	5973
7854	4635
296268	17919
888804	89595
2666412	806355
387814812	27684855

The former example shows an abbreviated method of using a multiplier which is frequently required in Practical Mathematics; the multiplier 5236, which is another constant number often used as a multiplier, allows the very same kind of simplification. The process of multiplying by 7854 consists in resolving that multiplier into 6000 + 1800 + 54, in order that we may multiply first by 6, then the result by 3, and then the last result by 3; for 18 times = 6 times $\times$ 3; and 54 times = 18 times $\times$ 3. In the latter example, the multiplier 4635 is resolved into 3000 + 1500 + 135, so that we compute for 3, 5 times 3, and 9 times 15.

Examp. Multiply 7385 by 365; also 9746 by 5286, and 836 by 892.

7385	9746	836
365	5286	892
36925	58476	6688
258475	467808	76912
2695525	467808	745712
	51517356	

In the first example, we first obtain the product by 5, *i. e.* divide ten times the multiplicand by 2, and place the figures as they arise in division; then we find the product by 35, which is just 7 times the line already found; lastly, to complete 36 times, we include the multiplicand in the addition.

In the second example, we resolve the multiplier into 6 + 480 + 4800; consequently, we first take 6 times, then 8 times the result, then a repetition of 8 times.

In the third example, let the product by 8 stand under the multiplicand, as if two ciphers were annexed to the latter; subtract the product by 8 from 83600, *i. e.* from the product by 100, and the remainder, extended two places to the right, will be the product by 92; because $100 - 8 = 92$.

It may perhaps be considered that some of the above modes of abbreviation are not such as would readily suggest themselves; and that therefore our exemplification of them is of no great practical utility: but we are sure that a little practice in trying to discover such contractions will induce a quickness of perception, enabling the pupil to perform his multiplications in very many instances with greater expedition and convenience than the ordinary method allows.

REDUCTION.

Numbers, in their application to the business of life, obtain names and values in addition to those of units, tens, &c. The number 236, for example, has its abstract or general value determined by the Numeration Table. But, while it always denotes 236 *units*, it may become a *concrete* number, by assuming an additional name, such as *inches*, *acres*, &c. Thus, 236 *inches* denotes 236 units of measure, each an inch long.

Now, a single unit of one denomination may be made equivalent to several units of an inferior denomination. Thus, 1 *mile* consists of 8 equal parts, called *furlongs*;

1 *shilling* is divided into 12 *pence;* 1 *pound* weight is equivalent to 16 *ounces;* &c.

In the Manual will be found Tables of such equivalents; and the pupil may be required to commit these Tables to memory, one by one, as preparation for the Exercises in Reduction, &c. may render necessary. For additional information respecting Concrete Equivalents, *see* Appendix, No. 1.

For *Definition* of Reduction, see Manual.

Examp. Let it be required to change 280 shillings into pence, and also into sovereigns.

As 1 shill. equals 12 pence, 280 shill. equal 280 times 12 pence; therefore, we multiply 12 by 280, or, for greater convenience, we multiply 280 by 12, and obtain 3360 pence as equivalent to 280 shill.

280 times
12 pence
3360 pence

Secondly, take 20 shill. out of 280 shill. as often as can be done, because every 20 shill. makes 1 sovereign. The division gives 14 times 20 shill.; therefore, 280 shill. are equivalent to 14 sovereigns.

20)280 shill.
14 sov.

Examp. Reduce 14160 farthings to crowns.

There are so many pence in the given sum as the number of times it contains 4 farthings; also, there are so many shill. in the sum as its equivalent number of pence contains 12 pence; lastly, there are so many crowns in the sum as its equivalent number of shill. contains 5 shill. Thus, by gradually lessening the numbers, and raising the denominations from farthings to crowns, we obtain 59 crowns as the answer. The accuracy of the process may be *proved* by reducing 59 crowns to farthings.

4)14160 farth.
12)3540 pence
5)295 shill.
59 crowns

These explanations prepare the way for the solution of the first fourteen Exercises in the Manual. Let the pupil be occa-

sionally required to test the correctness of his result, by reversing the problem.

Concrete quantities are of two kinds, Simple and Compound. A simple concrete quantity consists of only one denomination; as 23 miles. A compound concrete quantity consists of more than one denomination of the same kind; as 23 miles 6 furlongs.

We shall now exemplify Reduction involving compound quantities.

Examp. Reduce £8. 13*s.* 6*d.* to pence.

We employ the usual abbreviations £. *s.* *d.* to denote pounds, shillings, and pence, respectively. Now £8. are 8 times 20*s.* = 160*s.*, and 13*s.* added make 173*s.* Again, 173*s.* are 173 times 12*d.* = 2076*d.*, and 6*d.* added, make 2082*d.*

£8. 13*s.* 6*d.*
20
—
173*s.*
12
—
2082*d.* *Ans.*

Examp. Reduce 26794 farthings to £.

We first inquire how often 4 farthings is contained in the given quantity; the number of times is the equivalent number of pence. We next inquire how often 12 pence is found in the sum of pence, and thus we obtain, as an equivalent, a number of shillings with 2½*d.* over. Lastly, we inquire how often 20*s.* is found in the sum of shillings; and to the pounds and odd shillings thus obtained we annex the reserved pence.

4)26794*f.*
12)6698½*d.*
2,0)55,8*s.* 2½*d.*
Ans. £27. 18*s.* 2½*d.*

The above examples make it observable that a higher denomination is reduced to a lower by multiplication; and a lower to a higher by division. In both cases there is a *Reduction*, according to the ordinary acceptation of the term; in the one, a lessening of the *denomination*; in the other, a lessening of the *number*.

When the pupil has arrived at the 73rd Exercise he will require to be informed that in some cases Reduction is *indirect*, and involves both multiplication and division, as in the following example.

Examp. Convert 436 English ells into yards.

As there is not an exact number of yards in an English ell, we first reduce the given quantity to the most convenient denomination of which a yard and an ell severally contain an exact number; that is, we first change the English ells to quarters; and then we can directly convert the quarters into yards.

436 E. e.
5
4)2180 qrs.
Ans. 545 yds.

Otherwise, we might have found the number of yards in 436 *quarters*, that is, 436 ÷ 4 = 109 yds.; and then it would have remained to multiply 109 by 5, for there must be 5 times as many yards in 436 *E. e.* as in 436 *qrs.* It is, however, generally, preferable to make division the concluding process, on account of the remainders it may yield.

COMPOUND ADDITION.

For *Definition*, see Manual.

The various denominations of which money, weight, and measure, consist, are added, subtracted, &c. on the same principle as is employed with simple numbers. This may be illustrated by the two subjoined examples.

1. Add together 5239, 1476, 3587, and 2865.

2. Add together £52. 3*s.* 9*d.*, £14. 7*s.* 6*d.*, £35. 8*s.* 7*d.*, and £28. 6*s.* 5*d.*

Hund.	*Tens.*	*Units.*
52	3	9
14	7	6
35	8	7
28	6	5
131	6	7

Pounds.	*Shill.*	*Pence.*
52	3	9
14	7	6
35	8	7
28	6	5
130	6	3

The sum of the units is 27 units, which divided by 10 units gives 2 tens and 7 units; the 7 units being reserved, and the 2 tens carried; &c.

The sum of the pence is 27 pence, which, divided by 12 pence, gives 2 shill. and 3 pence; the 3 pence being reserved, and the 2 shill. carried to the column of shillings, whose sum is, in like manner, divided by 20 to get £.; &c.

In the first example, the amount is 131 hundreds, 6 tens, and 7 units, or 13167; in the second, the amount is 130 pounds, 6 shill. and 3 pence, or £130. 6*s*. 3*d*.

In the first example, the denominations are *uniformly* related, and *simple;* in the second, the denominations are *variously* related, and *compound*:—The denomination of hundreds is *ten* times the value of tens, and that of tens is *ten* times the value of units; whereas the denomination of pounds is *twenty* times the value of shillings, and that of shillings is *twelve* times the value of pence.

In the first of the two following examples the parts of a penny are added as farthings or fourths, thus: 2 farthings and 3 farthings make 5 farthings, and 1 farthing makes 6 farthings, = 1 penny and 2 farthings, or 1½*d*. We therefore write ½*d*. under the column of farthings, and carry 1 penny to the column of pence; &c.

In the second example, containing quantities in Troy weight, we first find the sum of the grains to be 55, which divided by 24 gives 2 dwt. and 7 grains; the sum of dwts., 31, divided by 20, gives 1 oz. and 11 dwt.; the sum of oz., 16, divided by 12, gives 1 lb. and 4 oz.; and, lastly the sum of lbs. is 27.

£.	s.	d.
29	12	6¼
36	15	11¾
85	9	7½
£151	18	1¼

lb.	oz.	dwt.	grs.
15	3	13	20
9	10	10	14
2	0	6	21
lb.27	4	11	7

COMPOUND SUBTRACTION.

The observations which have been made respecting Compound Addition are so obviously suggestive of the general illustration appropriate to the Subtraction of compound quantities, that here we shall merely furnish an example of the latter operation.

Examp. Subtract £138. 16*s.* 4½*d.* from £235. 13*s.* 9½*d.*

£235	13	9½
138	16	4½
£ 96	17	4¾

Beginning at the place of farthings, we say 2 farth. from 1 farth. cannot be taken; therefore (on the principle employed in Simple Subtraction), we add 4 farth. to the minuend and 1 penny to the subtrahend, and say 2 farth. from 5 farth. leaves 3 farth., and 5 pence from 9 pence leaves 4 pence; then, in order to subtract 16 shill., we add 20 shill. to the minuend and 1 pound to the subtrahend, and say 16 shill. from 33 shill. leaves 17 shill., and £139 from £235 leaves £96.

In cases where the shill., pence, &c. of the minuend are fewer than those in the subtrahend, the subtraction may be effected more easily by adding to the upper quantity the complement of the lower, or what it wants to make it of the next higher denomination. Thus, instead of saying 16*s.* from 33*s.*, we may say the complement of 16*s.* is 4*s.*, which added to 13*s.* make 17*s.*

COMPOUND MULTIPLICATION.

For this Rule no special illustration seems to be requisite, beyond one or two examples.

Examp. Multiply £95. 17*s.* 5¼*d.* by 3.

£95	17	5¼
		3
£287	12	5¼

Beginning at the place of farthings, we say 3 times 3 farthings is 9 farth., or 2 pence and 1 farth.; reserve ¼*d.* and carry 2*d.*; 3 times 5*d.* is 15*d.*, and 2*d.* carried makes 17*d.*, or 1*s.* and 5*d.*; reserve 5*d.* and carry 1*s.*; 3

times 17*s.* is 51*s.*, and 1*s.* carried makes 52*s.*, or £2 and 12*s.*; reserve 12*s.* and carry £2.; 3 times £95 is £285, and £2 carried makes £287.

The above is an example of the method applicable to the first 24 Exercises in the Manual. Those from the 25th to the 47th, inclusive, have composite multipliers, which may therefore be resolved into factors, as explained under the head of Simple Multiplication.

For the performance of the next set of Exercises, the usual procedure is to break up the multiplier into its thousands, hundreds, &c.; then to find the products of the given multiplicand by these portions separately; and lastly, to add the several products thus obtained. But many persons prefer to calculate the product by the undivided multiplier. We shall exemplify both these methods in finding the product of £23. 7*s.* 5*d.* by 365.

5 times =	£23	7	5	10	× 5 =	£116	17	1
60 times =	233	14	2	10	× 6 =	1402	5	0
300 times =	2337	1	8		× 3 =	7011	5	0
365 times . . . . . =						£8530	7	1

Otherwise.

5*d.* × 365 =	1825*d.* =	152*s.* 1*d.* =	£7	12	1
7*s.* × 365	= 2555*s.*	=	127	15	0
£23 × 365		=	8395	0	0
		Ans.	£8530	7	1

The latter will generally be found the more convenient method. But perhaps the most expeditious mode of using the particular multiplier 365, in Compound Multiplication, is to resolve it into 5 × 12 × 6 + 5; for the first product may be added in while the last multiplication is being performed.

COMPOUND DIVISION.

The first 18 Exercises in Compound Division propose to find Quotients properly so called:—They require to determine *how often* one concrete quantity is contained in another. When the dividend does not contain the divisor an exact number of times, the proper and entire quotient cannot be demanded from pupils unacquainted with the principles of fractional arithmetic; and therefore, in such a case, an undivided portion of the dividend may be allowed to stand over as a remainder.

Examp. Find how often 2*ro.* 34*po.* is contained in 22*ac.* 2*ro.* 10*po.*

As an expedient preparation, we reduce dividend and divisor to one denomination, that of *poles*. Then we find that the dividend contains the divisor 31 *times*, with a remainder of 76 poles, or 1ro. 36po. For the proper completion of the quotient we require to determine how often 114po. is contained in 76po.; but some familiarity with fractions is necessary to understand that this part of the quotient is 2 *thirds* of a time, and that the proper answer is 31⅔ times.

22*ac.* 2*ro.* 10*po.*
4
—
2*ro.* 34*po.* 90
40 40
— —
114*po.*) 3610*po.*

Ans. 31 *times.* *Rem.* 76*po.*

The remaining Exercises propose to find what divisor is contained a given number of times in a given dividend; but the given number of times is usually called the divisor, and the answer, the quotient.

Examp. What sum of money is contained 9 times in £237. 10*s.* 7½*d.*?

The 9th part of £237. or the highest number of £. contained 9 times in

9) £237. 10*s.* 7½*d.*

Ans. £26. 7*s.* 10*d.* *Rem.* 1½*d.*

£237, is £26, with £3 remaining; that remainder makes with the 10*s.* a dividend of 70*s.*, the 9th part of which is 7*s.*, with 7*s.* remaining; the next dividend is 7*s.* 7*d.*, or 91*d.*, the 9th part of which is 10*d.*, with 1*d.* remaining; the last dividend is 1½*d.*, or 6 farth., the 9th part of which is less than a farthing. Let, therefore, 1½*d.* stand over as an undivided remainder.

The proper answer would be determined by fractional operation to be £26. 7*s.* 10⅑*d.*, or £26. 7*s.* 10⅑*d.* ⅑*f.*

From the 35th Exercise to the 45th, the divisors are composite, and may be resolved into factors.

Examp. Required the 84th part of 243*da.* 15*ho.*

As 84 equals 7 × 12, we first find the 7th part of the given quantity, and then the 12th part of the 7th. At the end of the former division there remains 4 sec. undivided; and at the end of the latter division there is a remainder of 8 sec. *for each 7th part of the whole*, that is, a remainder of 56 sec., which with 4 sec. previously remaining makes 60 sec. or 1 min. as the entire remainder.

7)243*da.* 15*ho.*
12)34*da.* 19*ho.* 17*min.* 8*sec.* *Rem.* 4*sec.*
2*da.* 21*ho.* 36*min.* 25*sec.* *Rem.* 60*sec.*

The remainder divided by 84 would make the answer terminate properly with 25⅐ sec. And observe, it is optional whether the above division should be carried out to seconds; if we had ended with the denomination of minutes, the answer would have been 2 da. 21 ho. 36 min. *Rem.* 36 min., or 2 da. 21 ho. 36⅐ min.

For the performance of the 46th and most of the following Exercises, the process of Long Division is requisite.

Examp. Divide £1563. 13*s*. 5½*d*. into 347 equal parts.

347)£1563. 13*s*. 5½*d*.

Ans. Each part is £4. 10*s*. 1½*d*. *Rem.* 1*d*.
1388

The annexed operation proceeds on the same principle of successive division which we have previously exemplified; only, the remainders, reductions, &c. in this example, are not employed mentally, but noted down, to relieve the memory.

£175 *rem.*
20
3513*s*.
347
43*s*. *rem.*
12
521*d*.
347
174*d*. *rem.*
4
698*f*.
694
4*f*. *rem.*

In the Manual, a few Promiscuous Exercises are introduced before Simple Proportion; and some of them will be found to afford useful preparation for that Rule. All or most of them should now be carefully wrought by the pupil, as he will thereby increase his aptitude for the comprehension of subsequent departments of Arithmetic.

SIMPLE PROPORTION.

The Rule of Proportion teaches a method of comparing and computing quantities, which is often very convenient and useful; although questions in this rule might all be resolved independently of an acquaintance with its peculiar form. What Whiston says, in his edition of Tacquet's Euclid, is indeed true

—that if the doctrine of Proportion were withdrawn from Mathematics, that science should retain scarcely anything excellent or remarkable (Book V., Introd.). But the essential character of Proportion may be recognised in calculation without the artificial formality of its usual statement; and the teacher should frequently require the pupil to employ the demonstrative procedure of first principles in the solution of Proportion questions. We shall accordingly exemplify such procedure after we have explained the ordinary form of the Rule.

There are two ways of comparing one number with another, to determine their relative magnitude. We may determine that the one exceeds the other by a certain difference; or we may determine that the one contains the other a certain number of times. Thus, in comparing 45 with 15, we may pronounce the former to be 30 more than the latter, or we may pronounce the former to be 3 times as great as the latter. In either case we express a relation between the two numbers. But the second mode of consideration determines that relation which we call *Ratio.**

For *Definition* of Ratio, see Manual.

Now, if we equally increase or diminish the numbers 45 and 15, we shall not alter their relation as to difference; but we shall alter their relation as to ratio; 50 and 20 differ still by 30, but 50 is not 3 times 20.

Again: if we equally multiply or divide the numbers 45 and 15, we shall alter their relation as to absolute difference; but we shall not alter their relation as to ratio; 90 and 30 do not differ still by 30, but 90 is 3 times 30.

Ratio, then, is a relation independent of the absolute difference of numbers.

When the ratio of two numbers is equal to the ratio of two other numbers, the four numbers constitute

* *Number*, which is commonly declared to be "either a unit, or a multitude of units," may be defined by a reference to the notion of Ratio. "By *Number* we understand not so much a multitude of unities, as the abstracted ratio of any quantity to another quantity of the same kind, which we take for unity." —Newton's *Arith. Univ.*, by Raphson, p. 2.

what is called a *Proportion* or *Analogy*. Thus, the ratio of 45 to 15 is equal to the ratio of 36 to 12; for, in either case, the former number is 3 times the latter. Now, as the conventional sign expressive of ratio is two dots, we may write 45 : 15 = 36 : 12, and by these four terms we have constituted a Proportion. The statement signifies that 45 contains 15 as often as 36 contains 12; but it is commonly read thus, 45 *is to* 15 *as* 36 *is to* 12.

It has long been customary to employ four dots instead of the sign of equality; thus, 45 : 15 :: 36 : 12.

Now, suppose a question to be put in this form; 45 is to 15 as 36 is to — what number? We may easily supply the answer, from the consideration, that as 45 is 3 times 15, so 36 is 3 times 12. The object of the Rule of Simple Proportion is to propose such questions; that is, it furnishes three terms of a proportion, and requires to determine the fourth.

For *Definition*, see Manual.

We have thus indicated one method by which the solution of problems in Simple Proportion may be effected; that is, to divide the 3rd term by the quotient which arises from dividing the 2nd by the 1st. But the method usually practised is, to multiply the 2nd and 3rd terms together, and then to divide their product by the 1st. This is more convenient than the former; and it admits of easy demonstration. For, if instead of particular numbers we employ the letters a, b, and c, then, as every proportion signifies that $a \times c : a :: b \times c : b$, therefore, the product of the two inner or mean terms must always be equal to the product of the two outer or extreme terms; that is, $a \times b \times c = a \times c \times b$; and from this it is obviously deducible that the product of the means, divided by one extreme, gives the other extreme; so that the 4th term of any proportion is equal to the product of the 2nd and 3rd terms divided by the 1st. By special exemplification, we have 45 : 15 :: 36 : 12, the same as $15 \times 3 : 15 :: 12 \times 3 : 12$; and $15 \times 12 \times 3 =$

$15 \times 3 \times 12$, or $15 \times 36 = 45 \times 12$, or $15 \times 36 \div 45 = 12$.

The customary Rule for the solution of problems in Simple Proportion is the following:

Write, as the 3rd term of the analogy, that given quantity which is of the same nature as the required 4th term. If the 4th term is to be greater than the 3rd, let the greater of the two other given quantities be made the 2nd term; if less, let the less. Then multiply the 2nd and 3rd terms together, and divide their product by the 1st.

The first part of this Rule might be made less artificial in appearance, by saying, *Let the given quantities be so arranged that one of the two which are of like kind shall be to the other, as the remaining quantity is to the required quantity.*

Examp. If 18 gall. of rum cost £13., what cost 54 gall.?

Here it is evident that the given price of 18 gall. must be to the required price of 54 gall. as 18 to 54; because price varies directly as the quantity purchased; therefore,

18 gall. : 54 gall. :: £13 : the answer;
and £13 × 54 ÷ 18 = £39; consequently,
18 gall. : 54 gall. :: £13 : £39. *Ans.*

It is expedient, however, that the pupil should be frequently required to dispense with the customary form of statement, and to give a *demonstrative solution* rather than a *solution wrought according to a previously demonstrated form.*

Let us take the 1st, 3rd, 6th, and 7th Exercises in the Manual to furnish specimens of such demonstrative solution.

(1.) If 14 cwt. cost £42, then 1 cwt. costs £42 ÷ 14, and 20 cwt. cost £42 ÷ 14 × 20, or £42 × 20 ÷ 14. (3.) If 16 yds. cost 40*s.*, then 1 yd. costs 40*s.* ÷ 16, and 10 yds. cost 40*s.* × 10 ÷ 16. (6.) If 18*s.* is given for 24 lbs., then 1*s.* is given for 24 lbs. ÷ 18, and 12*s.* is given for 24 lbs. × 12 ÷ 18. (7.) If 6 men take 40 hrs., then 1 man would take 40 hrs. × 6, and 15 men would take 40 hrs. × 6 ÷ 15.*

* Questions like No. 7 are sometimes called Examples in Inverse Proportion, because the time is not *directly* as the number of men, but *inversely*. The time *decreases* as the number of men *increases*.

After the pupil has wrought the first nine Exercises in the Manual, it may be considered desirable to acquaint him with the expedient called cancelling, which is frequently available to simplify the process of multiplying by one number and dividing the product by another. For example, $27 \times 24 \div 18$ is equivalent to $27 \times 4 \div 3$, the dividend and divisor having their common factor, 6, cancelled in each; and it is also equivalent to $3 \times 24 \div 2$, the dividend and divisor having 9 cancelled in each. The expedient consists in reducing the divisor and either factor of the dividend by dividing each of them by some integral measure common to both. That the result is not affected by cancelling may be easily demonstrated.

For, in the first place, any number may be converted into an infinite variety of equal magnitudes, with different denominations. Thus, the concrete number £35 = 7 Five £ Notes = 700 shill. = 140 crowns = 100 sums of 7 shillings, &c. So also the abstract number 72 = 36 pairs = 6 dozen = 8 nines = 144 halves, &c.

Accordingly, when two numbers are related as dividend and divisor, and can be reduced to one common denomination expressed by simpler figures, the division may be facilitated by such reduction. Thus, 720 ÷ 48 = 60 dozen ÷ 4 dozen = 60 units ÷ 4 units = 15; so also 324 farthings ÷ 216 farthings = 81*d.* ÷ 54*d.* = 9 ninepences ÷ 6 ninepences = 3 sums (each 2*s.* 3*d.*) ÷ 2 sums (each 2*s.* 3*d.*) = 3 ÷ 2 = 1½.

Examp. If 56 lbs. of sugar cost 32*s.*, how many lbs. will cost 42*s.*?

The ratio of 56 lbs. to the answer must be the same as the ratio of 32*s.* to 42*s.*; therefore,

32*s.* : 42*s.* :: 56lbs. : *Ans.*

$$\frac{56 \times 42}{32} = \frac{7 \times 42}{4} = \frac{7 \times 21}{2} = \frac{147}{2} = 73\tfrac{1}{2} \text{ lbs. } \textit{Ans.}$$

The exercises from Numbers 10 to 21, inclusive, have a compound quantity as the third term. The 18th may be here wrought as an example, by demonstrative solution.

32 ac. &c. ÷ 1560 is the quantity worth £1.; therefore, the quantity worth £1430 is $\frac{32 \text{ ac. \&c.} \times 1430}{1560}$

Cancelling first 10, and then 13, gives

$$\frac{32 \text{ ac. \&c.} \times 11}{12} = 29 \text{ ac. } 3 \text{ ro. } 3 \text{ po. } \textit{Ans.}$$

When the first twenty-one Exercises have been wrought, some illustration will be necessary to enable the Pupil to deal with compound quantities occurring as first and second terms. In such instances, these terms cannot, in general, be conveniently employed in multiplication and division, till they are reduced to whole numbers of one denomination.

Let us select for solution the 22nd Exercise.

7 qrs. 3 bu. 2 pks. : 5 qrs. 6 bu. 3 pks. :: £154. : *Ans.*
or, 238 pks. : 187 pks :: £154.

$$\frac{187 \times 154}{238} = \frac{187 \times 11}{17} = 11 \times 11 = £121. \textit{ Ans.}$$

Let it be remembered, that when the three terms are stated, and the 1st and 2nd are reduced to quantities of one denomination, these two should be considered as divested of their concrete character, and the 3rd alone should retain the denomination in which it is given, or to which we may have found it expedient to reduce it. The two first terms imply a ratio, and ratio is always abstract. Thus, in the above example, we are required to diminish £154. in the ratio of 238 pks. : 187 pks. or in the ratio of 238 to 187. We retain the denomination of the 3rd term, because that is to be the denomination of the required 4th term.

As variety of illustration best promotes expertness in the management of Proportion questions, we refer to the Key in the Appendix for solutions of other Exercises in the Manual.

We shall conclude this article with a Table of such properties of arithmetical analogy as are most frequently found to be useful.

If a, c, m, and n, represent any four abstract numbers, and are such that $a : c :: m : n$, then it may be demonstrated that

$a : m :: c : n$	$a + c : c :: m + n : n$
$c : a :: n : m$	$a - c : c :: m - n : n$
$a + c : a - c :: m + n : m - n$	

Example.—For $a : c :: m : n$, put 36 : 27 :: 16 : 12;

36 : 16 :: 27 : 12	63 : 27 :: 28 : 12
27 : 36 :: 12 : 16	9 : 27 :: 4 : 12
63 : 9 :: 28 : 4.	

COMPOUND PROPORTION.

Ratio may be constituted by a comparison in respect of more than one particular. If we compare 24 lbs. and 32 lbs., we may determine that the ratio of the one to the other is that of 3 to 4, or $\frac{3}{4}$; but, if the comparison were between 24 lbs. Avoirdupois and 32 lbs. Troy, then, as the Troy lb. contains 5760 grains, and the Avoird. lb. 7000 such grains, the ratio must be that of 7000 grs. × 24 to 5760 grs. × 32, namely, a *compound* ratio.

A compound ratio is reduced to a simple ratio by performing the multiplication indicated in each term. But previous cancelling will often simplify the reduction. Thus, the above compound ratio is equal to $700 \times 3 : 576 \times 4 = 175 : 192$. The ratio, then, of 24 lbs. Avoird. to 32 lbs. Troy, in respect of 1 lb. of each, is that of 7000 to 5760; and their ratio in respect of the whole quantities, is that compounded of the ratios of 7000 to 5760 and 24 to 32; and by reduction to a simple ratio, we find that

24 lbs. Avoird. : 32 lbs. Troy :: 175 : 192.

For *Definition*, See Manual.

Examp. If 12 men reap 63 acres in 35 days; how many acres will 15 men reap, at that rate, in 20 days?

Here, 1 man takes 35 da. × 12, or 420 da. for 63 acres; 1 man takes 20 da. × 15, or 300 da. for the acres sought; therefore, If 63 ac. are reaped by one man in 420 days, how many acres will one man reap in 300 days? The question is now reduced to the form of Simple Proportion, and may be resolved by first principles, thus:

Quantity reaped in 1 day $= \frac{63 \text{ ac.}}{420}$; ditto reaped in

300 days $= \frac{63 \times 300}{420} = 9 \times 5 = 45$ ac. *Ans.*

The following is the usual method by statement:

$$\left.\begin{array}{l} 12 \text{ m.} : 15 \text{ m.} \\ 35 \text{ da.} : 20 \text{ da.} \end{array}\right\} :: 63 \text{ ac.} : 45 \text{ ac.}$$

$$420 : 300$$

The reason of the combined analogies may be thus stated. The given quantity, 63 ac., must be to the required quantity (as regards the number of reapers), as 12 m. : 15 m., and (as regards the number of days), as 35 da. : 20 da. Compounding these ratios, we obtain the simple analogy, 420 : 300 :: 63 ac. : 45 ac.

It is, however, expedient, after stating the terms, to arrange them as follows, in order to apply cancelling more conveniently:

$$\frac{63 \times 15 \times 20}{12 \times 35} = \frac{9 \times 5 \times 5}{1 \times 5} = 9 \times 5 = 45 \text{ ac.}$$

This procedure will be easily remembered from its similarity to that employed in Simple Proportion. As in Simple Proportion we multiply the 2nd and 3rd terms, and divide their product by the 1st; so in Compound Proportion we multiply the continual product of all the 2nd terms by the 3rd, and then divide by the continual product of all the 1st terms.

Examp. How many days will 10 men require to manufacture 47½ reams of paper, working 12 hours a day; when 15 men take 9 days, working 11 hours a day, to make 52½ reams?

$$\left.\begin{array}{l} 10 \text{ m.} : 15 \text{ m.} \\ 12 \text{ ho.} : 11 \text{ ho.} \\ 52\frac{1}{2} \text{ re.} : 47\frac{1}{2} \text{ re.} \end{array}\right\} :: 9 \text{ da.} : 11\frac{1}{2} \text{ da. } \textit{Ans.}$$

$$\frac{9 \times 15 \times 11 \times 190}{10 \times 12 \times 209} = \frac{9 \times 15 \times 11 \times 10}{10 \times 12 \times 11} = \frac{9 \times 15}{12} = \frac{45}{4}$$

The statement in the above solution, is thus determined. Days are required, therefore 9 days is the third term. Then we choose the numbers of men for comparison, and say, if 15 men take 9 days, 10 men

will take more days; so far, then, as the numbers of workmen are concerned, the answer will be greater than the third term in the ratio of 10 to 15. Next, we choose the hours per day for comparison, and say, if 11 hours a day require 9 days, 12 hours a day will require fewer days; so far, therefore, as the duration of daily work is concerned, the answer will be less than the third term, in the ratio of 12 to 11. Lastly, we choose for comparison the quantities of paper, and say, if $52\frac{1}{2}$ reams require 9 days, $47\frac{1}{2}$ reams will require fewer days; accordingly, so far as the quantities of paper are concerned, the answer will be less than the third term, in the ratio of $52\frac{1}{2}$ to $47\frac{1}{2}$, or, more conveniently, 209 quarters to 190 quarters.

The solution of the above question by first principles is as follows:

Given, $9 \times 15 \times 11$ hours for 1 man to make $52\frac{1}{2}$ reams; then, the 10th part of that time will suffice for 10 men to do the same; and reducing their hours to days, we have

$\frac{9 \times 15 \times 11}{10 \times 12}$ days for 10 men to make $52\frac{1}{2}$ reams;

$\frac{9 \times 15 \times 11}{10 \times 12 \times 209}$ days for 10 men to make 1 quarter ream;

$\frac{9 \times 15 \times 11 \times 190}{10 \times 12 \times 209}$ days for 10 men to make 190 ditto.

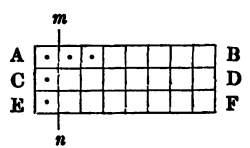
The teacher is recommended here, as in Simple Proportion, to require occasionally from the pupil solutions by first principles, as well as by the usual form of statement. Intellectual discipline will be thereby promoted, and the additional time thus expended will be well compensated by the acquirement of additional aptitude for calculation. For further exemplification of this Rule, see the Key in the Appendix.

VULGAR FRACTIONS.

Nature of a Fraction.

If we take any whole number, and divide it by a greater number, the quotient will of course be less than 1, and is called a *Fraction*. Thus, if we take the number 3, and divide it by 8, the quotient is $\frac{3}{8}$, which signifies $3 \div 8$, or the 8th part of 3. Such a quotient merely continues to represent the division from which it results, unless we can assign to the upper number, or dividend, some value which is convertible to a greater number of an inferior name. Thus, if by $\frac{3}{8}$ we signify the 8th part of 3 lbs., each lb. = 16 oz., then, the 8th part of 3 lbs. is the 8th part of 3 times 16 oz., and the quotient may be expressed by 6 oz. as well as by $\frac{3 \text{ lbs.}}{8}$. But if by $\frac{3}{8}$ we signify the 8th part of 3 farthings, then as we have no money denomination of less value than farthings, the quotient continues in the fractional or divisional form $\frac{3 f.}{8}$.

Further: As $\frac{3}{8}$ represents the 8th part of 3 units, it must also represent 3 times the 8th part of 1 unit; as may be demonstrated by means of the annexed diagram.



Let there be three contiguous rows of squares, A B, C D, E F, each row representing 1 unit divided into eight equal parts. Then, the three dotted squares from A towards B, are 3 eighth parts of 1 unit; and the three dotted squares cut off by *m n*, are 1 eighth of the whole diagram, that is, 1 eighth of 3 units; therefore 3 eighths of 1, and 1 eighth of 3, are equivalent.

This interpretation of a Fraction may be vindicated also thus:—Since 1 unit = 8 eighths of 1 unit; therefore, 3 units = 24 eighths of 1 unit; and, therefore, the eighth parts of these equals must be equal, or, 1 eighth of 3 units = 3 eighths of 1 unit.

For *Definition*, see Manual.

A Fraction then, is a proportionate part of a number; that is, it represents one of the equal parts into which its upper number is divided, or, one or more of the equal parts into which unity is divided. $\frac{2}{7}$ expresses 1 of seven equal parts into which 2 is divided, or 2 of seven equal parts into which 1 is divided.

When *unity* is the integer of which a fraction is understood to express a proportionate part, the lower term of the fraction is called the *Denominator*, as expressing the *denomination* of equal parts into which unity is supposed to be divided, and the upper term is called the *Numerator*, as expressing the *number* of equal parts taken.

Distinction of Fractions into Proper and Improper.

The *fractional form* may be assumed by a quantity whose value is unity, or higher than unity; because that form represents the relation of dividend and divisor, and any number may have its value represented by such relation. Thus, 8 is the quotient of $8 \div 1$, or of $16 \div 2$, &c., therefore 8 may be represented by $\frac{8}{1}$ or $\frac{16}{2}$, &c. Thus also $23\frac{4}{7}$ may be expressed in the purely fractional form $\frac{165}{7}$. Moreover, 1 may be expressed by $\frac{2}{2}$, $\frac{3}{3}$, $\frac{4}{4}$, &c.

A *Proper Fraction* is one whose numerator is less than its denominator, as $\frac{3}{4}$. An *Improper Fraction* is one whose numerator is not less than its denominator, as $\frac{4}{4}$, $\frac{12}{4}$, $\frac{10}{7}$. A *Mixed Number* consists of a whole number and a fraction, as $6\frac{3}{4}$.

An Improper Fraction is reduced to its proper form by performing the division which it indicates. Thus, $\frac{12}{4} = 3$; $\frac{10}{7} = 2\frac{3}{7}$.

A Mixed Number is reduced to an Improper Fraction, by converting the given whole number into that kind of parts of unity which the given fraction expresses, and then combining the two fractional expressions in one sum. Thus, $2\frac{3}{7} = \frac{14}{7} + \frac{3}{7} = \frac{17}{7}$.

A Whole Number is reduced to an Improper Fraction, by merely writing 1 as a denominator. Thus $6 = \frac{6}{1}$. Or we may take any number we please for the denominator, if we multiply it by the given number for a numerator. Thus, $11 = \frac{66}{6}$; also, $7 = 6\frac{7}{7}$.

The 1st and 2nd Sections in the Manual afford practice in these reductions.

Reduction of a Simple Fraction to its Lowest Terms, by the Greatest Common Measure.

The value of any given fraction may be represented by any other numerator and denominator having the same ratio between them as those of the given fraction have. Thus $\frac{3}{4}$ is equivalent to $\frac{6}{8}$, or to $\frac{9}{12}$, &c., because $3 : 4$ as $3 \times 2 : 4 \times 2$, or as $3 \times 7 : 4 \times 7$. It is here presumed that the pupil will recognise the fractional form as expressing the ratio of the two terms, because they constitute a dividend and divisor. Accordingly, if the terms of a fraction be both multiplied or both divided by any number, the value of the fraction is not changed.*

Now, it is often both practicable and expedient to divide the terms of a fraction by some number that will measure them both, and thereby lessen each term. Thus, $\frac{14}{70}$ may be changed into an equivalent fraction composed of lower terms; for if we divide both terms by 2, we obtain $\frac{7}{35}$, and if we further divide them by 7, we obtain $\frac{1}{5}$. To illustrate the equality of $\frac{1}{5}$ and $\frac{14}{70}$, suppose a school to consist of 70 boys; then, 42 of these boys make $\frac{14}{70}$ of the whole number, and make also $\frac{3}{5}$ of the whole number; for $\frac{1}{5}$ of the whole number is 14, and, therefore, $\frac{3}{5}$ of the whole number is 3 times 14, or 42.

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* Multiplying or dividing *both terms of a fraction* by the same number is, in effect, multiplying or dividing *the fraction* by *unity*. Thus $\frac{3}{4} \times \frac{7}{7} = \frac{21}{28}$; but $\frac{7}{7} = 1$; therefore $\frac{3}{4} = \frac{21}{28}$. This illustration may be given to the pupil after he has learned Division of Fractions.

A practised arithmetician will generally be able to determine, by a little inspection, what divisor is applicable for the reduction of a fraction to lower terms; but we shall here exhibit a process, by which we may directly ascertain the Greatest Measure common to two numbers; as such a divisor will at once reduce the terms of a fraction to the lowest exact numbers.

Suppose it is required to determine the greatest common measure of 1173 and 1518. The principal steps of the investigation* may be marked as follows:—

Whatever is the greatest common measure of 1518 and 1173,

i. e. of 1173 + 345 and 1173,
is also *g. c. m.* of 345 and 1173,
i. e. of 345 and 345 + 345 + 138,
and also *g. c. m.* of 345 and 138,
i. e. of 138 + 138 + 69 and 138,
and is also *g. c. m.* of 69 and 138,
that is, of 69 and 69 + 69;

but the greatest common measure applicable to this last form is evidently 69; therefore, 69 is the greatest common measure of 1173 and 1518; and on trial, it is found to be contained 22 times in the greater, and 17 times in the less.

The above process is virtually the same as the more convenient one exhibited in the margin; which consists in first dividing the greater of the given numbers by the less, then dividing the divisor by the remainder, and continuing to divide similarly until an exact divisor is found. That exact divisor is the greatest common measure of the given numbers.

When it is required to find the greatest measure common to three

```
1173)1518(1
     1173
     ----
 345)1173(3
     1035
     ----
 138) 345(2
      276
      ---
  69)138(2
      138
      ---
```

* For a more rigid demonstration, see La Croix, *Traité d'Arithmétique*, § 61; De Morgan's *Arithmetic*, 5th ed., p. 46; or Malcolm's *Arithmetic*, p. 118.

quantities, we take the third number and the greatest common measure of the first and second, as two quantities whose greatest common measure is that required.

Thus, to find the g. c. m. of 1173, 1518, and 713:—

The g. c. m. of the two first numbers is found, as above, to be 69; and the g. c. m. of 69 and 713 is found, as in the margin, to be 23. Therefore, 23 is the g. c. m. of the three given numbers.

69)713(10
690
—
23) 69(3
69
—

The pupil having performed the Exercises in Section 3rd, will then be prepared to reduce a fraction to its lowest terms, by employing the greatest common measure of the numerator and denominator.

Thus, to reduce to its lowest terms the fraction $\frac{136}{323}$, we find the g. c. m. of the numerator and denominator to be 17, which converts the fraction into $\frac{8}{19}$. Observe that as 19ths are a denomination 17 times as great as 323 rds., so 136 parts are 17 times as many as 8 parts; and, therefore, we have not altered the value of the fraction, in reducing it to lower terms.

Reduction to an Assigned Denominator.

From what has been developed of the nature of fractions, it will be found easy to convert numerators of one denomination into numerators of another.

Thus, to reduce 8 *twelfths* to an equivalent number of *eighteenths*. As equal fractions express equal ratios, therefore,

12 : 18 :: 8 : 12 eighteenths.

Otherwise; 8 *units* = 144 eighteenths of a unit; therefore, the 12th part of 8 units = 12 eighteenths.

Suppose it is required to reduce $\frac{7}{9}$ to 12ths:—

9 : 12 :: 7 : $9\frac{1}{3}$ twelfths.

This result shows the required fraction to be a *complex* one, viz. $\frac{9\frac{1}{2}}{12}$; and it must remain complex, if the denomination of 12ths is to be retained. The resolution to a simple fraction is effected by performing $9\frac{1}{2} \div 12$; but this will be better understood when we come to the consideration of fractional Division.

Reduction to the Least Common Denominator, by the Least Common Multiple.

For convenience in comparing fractions, we should have them expressed by one common denominator. Thus, it is not at once obvious whether $\frac{4}{9}$ or $\frac{5}{12}$ is the greater in value; but if each fraction is reduced to 36ths, then the expressions $\frac{1}{9}\frac{6}{6}$ and $\frac{1}{1}\frac{5}{2}$ indicate that $\frac{4}{9}$ is of higher value than $\frac{5}{12}$.

Any assemblage of fractions may be reduced to a common denominator; and it is often an easy matter to determine, by an inspection of the given denominators, for what common denominator they can be exchanged. If the given fractions are $\frac{4}{9}$, $\frac{5}{8}$, and $\frac{3}{8}$, it is easy to discern that each can be conveniently reduced to 72nds. If the Pupil, however, feel at a loss to decide in this way, he may always be sure that the continued product of the given denominators will constitute a common denominator, to which all the numerators may be conformed by the process pertaining to Section 5th.

But the continued product of the denominators does not generally supply the *least* common denominator that will suit the given fractions. Accordingly, a method has been investigated for determining the Least Common Multiple of several denominators, that is, the least number which contains each denominator an exact number of times.

The continued product of 6, 8, and 9, is 432, which is a multiple, therefore, of each of the factors; but 216 is also a multiple of each; so is 144, and so is 72.

Now, 72 is their *least* common multiple; that is, no number less than 72 admits of being exactly measured by 6, 8, and 9, severally.

Let us inquire then, how the least common multiple of several numbers may be determined; first, supposing it required to find the least comm. mult. of 18 and 24.

The greatest common measure of these numbers is 6, and, therefore, the smallest exact terms in which their ratio may be expressed are 3 and 4. Now, when we state 18 : 24 :: 3 : 4, we know that the product of the extremes equals that of the means; and as 3 and 4 are the lowest terms which can constitute the ratio of 18 to 24, therefore 18×4, or 24×3, must be the least common multiple of 18 and 24. The answer, therefore, is 72.

Again, suppose it is required to find the least comm. mult. of 18, 24, and 27.

The l. c. m. of the two first numbers is 72, as already found. Now, as 72 is not also a multiple of 27, the l. c. m. of the three numbers must be that of 72 and 27. Accordingly, the greatest comm. meas. of 72 and 27 is 9; and, therefore, the least terms of the ratio of 72 to 27 are 8 and 3; consequently 72×3, or 27×8, $= 216$, is the least comm. mult. required.

Note.—When, of the given numbers, one happens to be an exact multiple of another, that other may evidently be cancelled altogether. If, for example, the l. c. m. of 4, 6, 8, and 9, be required, it is manifest that whatever is a multiple of 8, must be also a multiple of 4; therefore, the l. c. m. of 6, 8, and 9, will be that of the four given numbers.

As a practical rule, then, for finding the least common multiple of two or more whole numbers:—Determine, by observation or trial, the greatest common measure of any two of the given numbers; divide one of them by it, and multiply the other by the quotient; the result is the l. c. m. of the two selected numbers. Then take that result and another of the given numbers, and repeat the process, &c.

Thus, to find the least comm. mult. of 25, 36, 40, and 32 :—

5)25, 40	8)8 × 25, 32	4)25 × 32, 36
8	25	25 × 8 × 36 = 7200. *Ans.*

To apply this process to its usual purpose, let it be required to represent the following fractions by equivalent ones having a common denominator : $\frac{7}{10}$, $\frac{5}{18}$, $\frac{4}{15}$, $\frac{1}{12}$.

2)10, 18	15)9 × 10, 15	6)9 × 10, 24
9	1	9 × 10 × 4

The l. c. m. of the denominators is 360; therefore, according to Section 5th,

$\frac{7}{10} = \frac{252}{360}$ $\quad$ $\frac{4}{15} = \frac{96}{360}$

$\frac{5}{18} = \frac{100}{360}$ $\quad$ $\frac{1}{12} = \frac{30}{360}$

Addition and Subtraction of Vulgar Fractions.

As we cannot tell the sum or the difference of 23 crowns and 43 pence, before reducing the two quantities to some common denomination; so we cannot tell the sum or the difference of 3 *fourths* and 5 *sixths*, before reducing the fractions to a common denominator. And when such reduction is performed, it is manifest that as the denominators express merely the *denomination* of the quantities, the change made by the addition or subtraction of the fractions applies to their numerators only.

Thus, if it be required to find the sum of $\frac{7}{10} + \frac{5}{18} + \frac{4}{15} + \frac{1}{12}$; we obtain, from the last example, 252 + 100 + 96 + 195 = 643, the sum of the proper numerators; and $\frac{643}{360} = 1\frac{283}{360}$. *Ans.*

In like manner, if it be required to find the difference of $\frac{3}{8}$ and $\frac{1}{7}$, we take the difference of $\frac{21}{56}$ and $\frac{8}{56}$, which is $\frac{13}{56}$. *Ans.*

When any of the aggregate parts proposed for addition are whole or mixed numbers, add the fractions by themselves, and afterwards combine their sum with that of the given integral quantities.

We subjoin two Subtraction examples, in solving which we employ an expedient requiring illustration.

Find the difference between 17 and $2\frac{1}{16}$, and also between $56\frac{3}{8}$ and $28\frac{1}{7}$.

In the first example, as there are no 26ths expressed in the minuend, conceive $\frac{1}{16}$ to be added to the minuend, and the equivalent 1 to be added to the subtrahend; then the upper number is $17\frac{14}{16}$, and the under $3\frac{1}{16}$; the subtraction is then easy.

17	$56\frac{3}{8}$
$2\frac{1}{16}$	$28\frac{1}{7}$
$14\frac{13}{16}$	$27\frac{2}{56}$

In the second example, we cannot subtract $\frac{1}{56}$ from $\frac{3}{56}$; but if we increase the minuend by $\frac{5}{56}$, and the subtrahend by 1, we can easily take $29\frac{1}{56}$ from $56\frac{7}{56}$.

Multiplication of Vulgar Fractions.

Suppose it is required to multiply $\frac{2}{5}$ by 4. Then, from what has been shown of Addition, it must be evident that $\frac{2}{5} \times 4$, or $\frac{2}{5}$ taken 4 times, is $\frac{8}{5}$. Here the operation has been made to affect only the numerator. But it is manifest, from the product $\frac{2 \times 4}{5}$ being equal to $\frac{8}{5}$, that dividing the denominator will be, in some instances, a preferable mode of multiplying a fraction by a whole number.

Suppose it is required to multiply $\frac{9}{10}$ by $\frac{1}{3}$. The multiplicand is to be taken $\frac{1}{3}$ of once, or it is to be divided into 3 equal parts, and one of these is to be taken. Accordingly, we shall have divided $\frac{9}{10}$ by 3, if we make its denominator 30ths, so that $\frac{9}{10} \times \frac{1}{3} = \frac{9}{30}$; or we shall have divided $\frac{9}{10}$ by 3 if we make its numerator the 3rd part of 9, so that $\frac{9}{10} \times \frac{1}{3} = \frac{3}{10}$. It appears, then, that dividing the numerator will be, in some instances,

a preferable mode of multiplying a fraction by a primitive fraction.*

Suppose it is required to multiply $\frac{2}{5}$ by $\frac{1}{9}$. We are required to take 4 times the 9th of $\frac{2}{5}$; and as $\frac{1}{9}$ of $\frac{2}{5}$ is $\frac{2}{45}$, therefore 4 times the 9th must be $\frac{4}{45}$ or $\frac{1}{11}$.

It hence appears that the multiplication of fractions is effected when we have placed the product of the given numerators over that of the given denominators.

Thus, $\frac{2}{5} \times 4 = \frac{2}{5} \times \frac{4}{1} = \frac{8}{5}$; also $\frac{9}{10} \times \frac{1}{5} = \frac{9}{50}$; also $\frac{2}{5} \times \frac{1}{7} = \frac{2}{35}$.

The numerators and denominators of fractions in multiplication, often admit the application of cancelling, on the principle of reduction to lower terms; thus,

$$\frac{9}{10} \times \frac{25}{27} = \frac{25 \times 25}{10 \times 27} = \frac{1 \times 5}{2 \times 9} = \frac{5}{18}.$$

The word *of* is often used between fractions, as an equivalent for the sign of multiplication, in which case the multiplier precedes the multiplicand. Thus, $\frac{1}{3}$ *of* $\frac{7}{10} = \frac{7}{10} \times \frac{1}{3}$. It seems needless to give such a form as $\frac{1}{3}$ of $\frac{7}{10}$ the name of Compound Fraction, as many writers on Arithmetic have done.

When Mixed Numbers are proposed for multiplication, the general mode of dealing with them is, first to convert them into Improper Fractions, and then to employ the operation already illustrated. But when the integral part of a given factor is large, it is generally

* A *primitive fraction* is one whose numerator is 1, as $\frac{1}{3}$. It is the reciprocal of a whole number; thus, $\frac{1}{3}$ is the reciprocal of 3; $\frac{1}{5}$ is the reciprocal of 5.

The multiplication of any number by a primitive fraction is resolvable into pure division. "*Multiplication*, properly so called," says Newton, "is that which is made by Integers, as seeking a new quantity, so many times greater than the multiplicand as the multiplier is greater than unity. But, for want of a better word, *Multiplication* is also made use of in Fractions and Surds, to find a new quantity, in the same *ratio* (whatever it be) to the multiplicand as the multiplier has to unity."—*Arithm. Univers.*, translated by Raphson.

expedient to find the entire product by means of two partial products; thus, for the 16th Exercise—

$$8375\tfrac{4}{11}\tfrac{4}{11} \times 435 = 8375 \times 435 + \tfrac{4}{11}\tfrac{4}{11} \times 435.$$

$$8375 \times 435 = 3643125$$

$$\tfrac{4}{11}\tfrac{4}{11} \times 4\tfrac{4}{11} = \frac{8 \cdot 8 \times 8}{8} = 304\tfrac{4}{11}$$

Ans. $3643429\frac{4}{11}$

Note.—It will be useful for the Pupil to recognise readily, that when a fraction is multiplied by a number equal to the denominator, the product is equal to the numerator; thus, 4 times $\frac{3}{4}$ is 3.

Division of Vulgar Fractions.

Required to divide $\frac{4}{7}$ by 3.

If by this it is meant to divide $\frac{4}{7}$ into 3 equal parts, then each part is just $\frac{1}{3}$ of $\frac{4}{7}$, or $\frac{4}{21}$.

If the meaning is, to find how often 3 is contained in $\frac{4}{7}$, then as 3 is contained in 6 exactly 2 times, therefore in the 7th part of 6 it is contained the 7th part of 2 times, or $\frac{2}{7}$ of a time.*

In the former case the answer is $\frac{1}{3}$ of *a unit*, in the latter case $\frac{2}{7}$ of *once*.

But the best general form of illustrating fractional division is by conceiving the dividend and divisor to have a common denominator equal to the product of the given denominators.

Thus, $\frac{7}{8} \div \frac{1}{3} = \frac{21}{24} \div \frac{8}{24} = 21 \div 9 = 2\frac{1}{3}$.

$\frac{4}{8} \div \frac{7}{8} = \frac{44}{88} \div \frac{49}{88} = 40 \div 42 = \frac{40}{42}$.

In these examples, it is evident that there is no necessity to express the common denominator, just as there is no necessity to express the common denomina-

---

* "Division is properly that which is made use of for integer or whole numbers, in finding a new quantity so much less than the dividend, as unity is than the divisor."—Newton's *Arithm. Univers.*, by Raphson.

tion of *sixpences* or *pence*, to which we might reduce £20. 10*s.* and 12*s.* 6*d.* in order to find how often the latter sum is contained in the former.

Hence, the rule for fractional division may be thus stated:—

Multiply the dividend by the reciprocal of the divisor, *i. e.* by the divisor inverted.

Accordingly, $\frac{5}{8} \div \frac{7}{8} = \frac{5}{8} \times \frac{8}{7} = \frac{5 \times 4}{8 \times 7} = \frac{20}{56}$.

When Mixed Numbers are given, and the integral portion is large, we may multiply the dividend and divisor by some common multiple of the denominators, which will cancel the fractions, and then divide the one result by the other.

Thus, to divide $943\frac{3}{15}$ by $47\frac{3}{10}$.

$47\frac{3}{10}$	$943\frac{3}{15}$
10	15
473	14147
3	2
1419)	28294

$19\frac{1}{1}\frac{8}{4}\frac{2}{1}\frac{8}{9} = 19\frac{2}{9}$ *Ans.*

In this example we have multiplied each given number by the least common multiple of 10 and 15, viz. 30, which, for convenience, we have applied to the divisor by the two factors 10 and 3, and to the dividend by the two factors 15 and 2.

Recognising the fractional form itself as representative of division, the pupil will now experience no difficulty in reducing what are called complex fractions to their simplest form.

For example, $\frac{2\frac{1}{2}}{7} = 2\frac{1}{2} \div 7 = 14 \div 35 = \frac{14}{35} = \frac{2}{5}$.

So also, $\frac{\frac{1}{2}}{1\frac{2}{5}} = \frac{1}{2} \div 1\frac{2}{5} = 2 \div 11 = \frac{2}{11}$.

These fractions sometimes arise as remainders from division, which is the case in the latter of the two subjoined methods of dividing $1375\frac{1}{5}$ by 12.

$$\begin{array}{rr} 12 & 1375\frac{1}{5} \\ 5 & 5 \\ \hline 60) & 6876 \\ & \hline & 114\frac{36}{60} \end{array} \qquad \begin{array}{r} 12)1375\frac{1}{5} \\ \hline 114\frac{8}{5} \end{array}$$

By the former method the remaining fraction is $\frac{36}{60} = \frac{3}{5}$. By the latter method it is

$$\frac{7\frac{1}{5}}{12} = 7\frac{1}{5} \div 12 = 36 \div 60 = \frac{3}{5}.$$

The latter method is often preferable, when the divisor is a whole number.

Reduction &c. of Concrete Fractions.

Let it be required to reduce $£\frac{3}{14}$ to pence:—

As £. are reduced to pence by multiplying by 20 and 12 successively, or at once by 240, we therefore multiply the given fraction thus—

$\frac{3}{14} \times \frac{20}{1} \times \frac{12}{1}$ or $\frac{3}{14} \times \frac{240}{1} = \frac{3 \times 120}{7} = 85\frac{5}{7}d.$ *Ans.*

Let it be required to reduce $85\frac{5}{7}d.$ to the denomination of £.:—

Pence are reduced to £. by dividing successively by 12 and 20, or multiplying successively by $\frac{1}{12}$ and $\frac{1}{20}$. Accordingly, $\frac{85}{7} \times \frac{1}{12} \times \frac{1}{20} = \frac{50}{7 \times 240} = £\frac{5}{14}$. *Ans.*

If it be required to convert $1\frac{1}{4}$ lb. into an equivalent quantity expressed in the denomination of cwt.:—

Lbs. become cwt. by division by 28 and 4; consequently, $\frac{1}{4} \times \frac{1}{28} \times \frac{1}{4} = \frac{1}{112}$ cwt. *Ans.*

Suppose we wish to express $\frac{5}{14}$ of a guinea as an equivalent fraction of a £.:—

Guineas are reduced to £. by first multiplying by 21

and then dividing by 20; or by multiplying by $\frac{1}{20}$, for each guinea is $\frac{21}{20}$ of a £. Accordingly,

$$\frac{5}{12} \times \frac{21}{20} = \frac{1}{8} \times \frac{3}{4} = £\frac{3}{8}$$

It is manifest from the above examples, that if the pupil has become familiar with fractional multiplication and division, the teacher will find little difficulty in enabling him to apply to concrete fractional quantities the principles of the common Rule of Reduction. Let the pupil be prompted to consider, with reference to a fractional part of a denomination, how he would reduce an integer of the same denomination; and it only remains for him to attend to the peculiar form in which multiplication or division is effected with fractions. In some books of Arithmetic, the Reduction of Concrete Fractions precedes the consideration of fractional multiplication. This is not a natural nor an expedient arrangement.

When the first 18 Exercises of Section 12th are performed, the following illustration may be entered upon.

When one quantity is proposed in several denominations, to be reduced to an equivalent single denomination, either of two methods may be adopted.

Let it be proposed to express 12*s*. 10½*d*. in the denomination of £.:—

1*st Method.* 12*s*. 10½*d*. = 309 *hf. d.*; and each halfpenny is the 480th of £1.; therefore, 12*s*. 10½*d*. = $\frac{309}{480}$ of £1. = £$\frac{643}{1000}$. *Ans.*

2*nd Method.* As each penny is $\frac{1}{240}$*s*., therefore 10½*d*. = $\frac{10\frac{1}{2}}{12} = \frac{21}{24} = \frac{7}{8}$*s*.; also 12*s*. 10½*d*. = 12$\frac{7}{8}$*s*. Moreover, each shilling is £$\frac{1}{20}$, therefore

$$12\tfrac{7}{8}s. = \frac{12\frac{7}{8}}{20} = £\tfrac{643}{1000}. \quad Ans.$$

The latter method will be preferred by a practised arithmetician. But both may be required from the pupil in his solutions of Exercises 19 to 27.

When the integer values equivalent to given concrete fractions are required, the process is just the same as for the elementary Rule of Compound Division. Thus, £$\frac{5}{7}$ = £5 ÷ 6 = 16*s*. 8*d*.; so also 5$\frac{4}{7}$ hours = 5 ho. + 4 ho. ÷ 7 = 5 ho. 34 min. 17$\frac{1}{7}$ sec.

Accordingly, Ex. 28 to 34 will be found exceedingly easy of

solution. And an inspection of the subjoined examples will be a sufficiently instructive preparation for the solution of the remaining Exercises of Section 12th.

Let it be required to find the sum and the difference of $1\frac{3}{8}$ qrs. and $\frac{3}{10}$ cwt. :—

$1\frac{3}{8}$ qrs. = 1 qr. + 8 qrs. $\div$ 9 = 1 qr. 24 lb. $14\frac{2}{9}$ oz.
$\frac{3}{10}$ cwt. = 3 cwt. $\div$ 10 = 1 5 $9\frac{1}{5}$

Ans. { Diff. 0 19 $4\frac{4}{45}$
Sum 3 2 $7\frac{7}{45}$

Multiply £3. 16*s*. $5\frac{1}{2}$*d*. by $11\frac{1}{3}$:—

$11\frac{1}{3}$

5)7 12 $11\frac{1}{3}$ = 2*ce* the given sum.

1 10 $7\frac{1}{15}$ = $\frac{1}{3}$ of the given sum.
42 1 $1\frac{1}{3}$ = 11 times ditto.

£43 11 $8\frac{8}{15}$ *Ans.*

Find the 8th part of 3 ro. 21 po. 26 sq. yds. $3\frac{1}{7}$ sq. ft.

8)141 po. 26 sq. yds. 3 sq. ft. $41\frac{1}{7}$ sq. in.

17 22 1 $117\frac{8}{7}$ *Ans.*

In this last example, the remainder 5 po. is multiplied by $30\frac{1}{4}$, and 26 yds. are added; then, division by 8 leaves a remainder of $1\frac{1}{4}$ yd., which multiplied by 9, and 3 being added, the division by 8 leaves a remainder of $6\frac{1}{2}$ ft., &c.

How often is $3\frac{3}{5}$ sec. contained in $\frac{4}{15}$ hour?
$\frac{4}{15}$ ho. $\times \frac{60}{1} \times \frac{60}{1} = \frac{14400}{15}$ sec., which divided by $\frac{18}{5}$ sec., or multiplied by $\frac{5}{18}$, gives $\frac{4800}{15} = 311\frac{7}{15}$ times. *Ans.*

In concluding the department of Vulgar Fractions, we may remark, that though they may appear, under a superficial consideration, to have little practical utility, they really are of great service in commercial affairs. A knowledge of them is a key to the general principles of calculation. It is recommended, therefore, that the pupil should be well practised in fractional computation. The Promiscuous Exercises in Section 13 will furnish a good test of his acquaintance with this subject.

DECIMAL FRACTIONS.

Conversion of Vulgar into Decimal Fractions.

We have shown that any Vulgar Fraction may be converted into an equivalent one having any proposed Denominator. We may, therefore, convert any given fraction into one that shall have for its denominator 10, or 100, or 1000, &c., that is, into a fraction having for its denominator 1 with a cipher or ciphers annexed. Such reduction produces what are called *Decimal Fractions*, from their numerators denoting so many *tenths*, or so many *tenths of tenths*, &c.

Examp. What numerator must $\frac{17}{32}$ assume, that its denominator may be converted into 10, into 100, into 1000, into 10000, into 100000?

32	:	17	::	10	:	$5\frac{12}{32}$ tenths. *Ans.*
32	:	17	::	100	:	$53\frac{4}{32}$ hundredths. *Ans.*
		&c.				&c.

It will be found more convenient, however, to effect the required changes in the following manner:—

The given fraction is equal to $\frac{17}{32} = \frac{1700}{3200} = \frac{17000}{32000}$, &c.

Dividing each term of the first of these by 32, gives the value in 10ths; dividing each term of the second by 32, gives the value in 100ths; dividing each term of the third gives the value in 1000ths, and so forth.

Now it will be found that an exact numerator will not be obtained, till we put the given fraction $= \frac{170000000}{320000000}$; dividing then each term by 32 gives $\frac{5312500}{10000000}$, which is a *pure* decimal, equivalent to $\frac{17}{32}$.

It appears, then, that to convert a Vulgar Fraction into a Decimal, we should annex one cipher after another to the given numerator, to make it admit of division by the given denominator, and the quotient figure arising from the annexation of the first cipher, will denote so many tenths; from the second, so many hundredths; from the third, so many thousandths; and

so forth. Now, we may stop the division when we please. If, for instance, we stop before dividing thousandths, we shall obtain, as the decimal form of $\frac{7}{13}$, the mixed fraction $53\frac{1}{8}$ *hundredths*, but it is generally expedient to carry out the division to its exact limit, when that is attainable within eight or nine decimal places.

It most frequently happens, however, that the division will never reach an exact termination.* Thus, let it be required to convert $\frac{5}{13}$ into the decimal form :—

The division, as will be seen in the margin, yields a numerator which will consist of a perpetual recurrence of the figures 384615 ; we cannot, therefore, obtain a pure decimal ; but this imperfection is of no consequence in the practical applications of Arithmetic, as a portion of the interminate numerator will be found a sufficiently accurate expression of the value of $\frac{5}{13}$. If we take the first four figures, $\frac{3846}{10000}$, we neglect only $\frac{5}{13}$ of the *ten thousandth* part of a unit. If we take the first six figures, $\frac{384615}{1000000}$, we neglect only $\frac{5}{13}$ of the *millionth* part of a unit, or less than the *two-millionth*.

```
13)500000000
  ----------
  38461538
```

It may now be observed, that if the first or left-hand figure of the numerator of a decimal fraction be always the place of *tenths*, the denominator will be thereby determined to consist of 1, with a cipher for each figure of the numerator. Accordingly, $\frac{6}{1000}$, as it consists of $\frac{9}{10} + \frac{8}{100} + \frac{6}{1000}$, will be represented by $\frac{986}{1000}$, in order that the number of ciphers in the denominator may equal the number of figures in the numerator. As this form of expression has been agreed upon, we do not require to write the denominator of a decimal fraction ; only, we must have some sign upon the numerator to prevent its being mistaken for an integral quantity.

* It never reaches an exact termination unless the divisor can be divided down to unity by 5 or 2.

The sign customarily employed for this purpose, is an elevated point preceding the place of tenths; thus, ·5 denotes $\frac{5}{10}$, and ·025 denotes $\frac{25}{1000}$ or $\frac{5}{200}$.

Accordingly, in converting Vulgar into Decimal Fractions, the correct form of operation is that exemplified in the margin, by the reduction of $\frac{1}{16}$ to the equivalent decimal 625 *ten-thousandths*. In this particular example, the dividend may be named either 1 *unit*, or 10 *tenths*, or 100 *hundredths*, &c., and a cipher must occupy the quotient place of tenths, to give the following figure its proper local indication as hundredths.

16)1·0000
·0625

Ciphers on the left, in a decimal numerator, affect its value; on the right they do not. Thus, ·05 is not = ·5, because ·05 = $\frac{5}{100}$, whereas ·5 = $\frac{5}{10}$; but ·5 is = ·50, because $\frac{5}{10} = \frac{50}{100}$.

When for convenience we retain only an approximate value of a decimal, and the first two of the figures which we reject are greater than 50, we should increase by 1 the last of the figures which we retain; because, then the excess above accuracy will be smaller than the defect below accuracy would have been. Thus, as an approximate value for ·46274, it is evident that ·463 is nearer the truth than ·462 would be.

Before concluding this section, we shall exemplify a convenient method of extending certain decimal quotients, without the trouble of division.

Suppose we are required to convert $\frac{1}{17}$ to its decimal equivalent:—

Dividing 1 by 17, we obtain ·0588235294$\frac{2}{17}$ as the value of $\frac{1}{17}$; therefore $\frac{2}{17}$ = ·1176470588$\frac{4}{17}$, obtained by doubling the quotient; and hence, by substitution of decimals, we have $\frac{1}{17}$ = ·0588235294117647, &c.

It is, however, rarely necessary to employ in calculation such a degree of decimal precision as this method so rapidly attains; and when absolute accuracy is required, fractions of the vulgar form are generally preferable to large decimals.

Conversion of Decimal into Vulgar Fractions.

The pupil having wrought the exercises in Sect. 1st, may then learn to reduce decimal fractions to the form of vulgar fractions. This, of course, may be accomplished by depriving the given decimal of its point, and writing it over its understood denominator. Let the fraction then be reduced, if possible, to lower terms.

Examp. Reduce ·825, and 24·04, to the vulgar form.

$\frac{825}{1000} = \frac{33}{40}$ *Ans.* $\qquad 24\frac{4}{100} = 24\frac{1}{25}$ *Ans.*

When the pupil has arrived at the 13th Exercise of Section 2nd, he should be made acquainted with the usual notation of *Recurring* or *Repeating* Decimals, that is, of Decimals in which one or more figures perpetually recur, as ·333, &c., ·2323, &c., ·05374374, &c.

Interminate Decimals of this form are usually indicated by head points. If a single figure recur, it is written once, with a point over it; and if a period of figures recur, it is called a circulating decimal, and is denoted by two points over the first and last figures of the period, respectively. Thus, ·444, &c., is written $\cdot\dot{4}$; also, ·62727, &c., is written $\cdot\dot{6}\dot{2}7$; and $7\cdot630630$, &c., is written $7\cdot\dot{6}3\dot{0}$.

Recurring Decimals are convertible into finite fractions, by writing 9, 99, &c., instead of 10, 100, &c., as the denominator. Thus, $\cdot\dot{7}\dot{6} = \frac{76}{99}$. A demonstrative illustration of this may be given in the following manner:—

Since $\quad 7600 = 76 \times 99 + 76,$
therefore $\frac{7600}{99} = 76 \quad + \frac{76}{99}.$

Hence, any number consisting of two digits, if divided by 99, will yield the same digits for the first two decimal figures of the quotient, and the same digits for a remainder; so that a continuation of the division will

produce a perpetual recurrence of the same two digits in the quotient.

In like manner it may be shown, that any other number, if divided by as many *nines* as it contains digits, will yield these digits as a decimal quotient, together with the same digits as a remainder. The following is another form of illustration :—*

$\frac{1000}{1000}$ of $\cdot\dot{3}\dot{2}\dot{5} = \cdot 325325$ &c.

$\frac{1}{1000}$ of do. $= \cdot 000325$ &c.

By Subtraction, $\frac{999}{1000}$ of do. $= \cdot 325$;

Therefore, 999 times $\cdot\dot{3}\dot{2}\dot{5} = 325$; or, $\cdot\dot{3}\dot{2}\dot{5} = \frac{325}{999}$.

When decimal figures precede the recurring part, a complex fraction will arise, which is treated as follows :—

Examp. Convert $1\cdot 048$, and $\cdot\dot{3}0\dot{6}$ to the vulgar form.

$1\cdot 04\frac{2}{3} = 1\frac{48}{100} = 1\frac{4\,8}{9\,9\,0} = 1\frac{1}{2\,0\,5}$ *Ans.*

$\cdot 3\frac{0\,6}{9\,9} = \frac{33}{10} = \frac{1\,0\,1}{3\,3\,0}$ *Ans.*

Addition and Subtraction of Decimals.

Addition and Subtraction of Decimals are similar to

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* See an additional illustration of recurring decimals, under the head of *Equirational Series.*

With respect to recurring decimals, it may be said generally, that they are not so convenient in computation as their corresponding vulgar fractions, unless we are permitted to use them approximately, like other decimals.

"*Complete* operations with circulating decimals," says De Morgan, "are *ignes fatui* which have led many an arithmetical writer astray: . . . . . Attempts to compute with interminable fractions are things on which real business will never waste a thought."—*Notices of Arithmetical Books*, pp. 69, 75.

those of Integers; but as the constant commencement of integers is at the place of units, or at the right hand, whereas the constant commencement of decimals is at the place of tenths, or at the left hand, we shall always have a full column on the left in decimals, whether the other columns be partial or not; whereas the reverse obtains with integers.

Examp. Required the sum and the difference of ·46 and ·366.

	·46		·46
	·366		·366
Sum	·826	Diff.	·094

In these examples, the principle of arrangement is that of placing all the tenths in one column, all the hundredths in another, &c. This will make all the decimal points fall into one column. Vacant places on the right may be conceived as being supplied with ciphers, because ciphers on the right of a decimal do not affect its value. The addition, or subtraction, then proceeds just as with integers; because in decimal fractions, as in whole numbers, the local values increase from right to left by a tenfold proportion.

Examp. Add, decimally, $2\frac{5}{12}$, $27\frac{7}{22}$, $5\frac{1}{3}\frac{2}{7}$, and $\frac{2}{5}$.

We have reduced the vulgar fractions to approximate decimals, with two different degrees of accuracy. The first answer is about three 100000th parts of a unit above the truth; the second about a ten-millionth part of a unit below it.	2·4167	2·4166667
	27·3182	27·3181818
	5·3243	5·3243243
	·375	·375
	35·4342	35·4341728

It is, however, generally expedient, in finding approximate answers, to make the operation extend one or two figures beyond that at which we mean the answer to terminate.

Examp. Subtract $\cdot\dot{4}\dot{2}\dot{6}$ from $20\frac{1}{13}$, securing five decimal places.

$20\cdot0344827 - \cdot4262626 = 19\cdot60822+$ *Ans.*

The teacher may easily extend the number of Exercises in Addition and Subtraction of Decimals by selecting from Sections 8 and 9 of Vulgar Fractions, and requiring the pupil, first, to reduce the given fractions to decimals; then, to add or subtract as is required; and lastly, to reduce the given answer to decimal form, and compare it with the answer previously obtained.

Multiplication and Division of Decimals.

The 10th part of a number may be represented, b so placing the decimal point between two of its figures, as to make what before was *tens* to represent *units*, what was *units* to represent *tenths*, what was *tenths* to become *hundredths*, &c. This will evidently be effected by shifting the point from its place in the given number to the other side of the figure which precedes it. Thus, $7\cdot6 \div 10 = \cdot76$, that is, the 10th part of 7 *units* and 6 *tenths* is 7 *tenths* and 6 *hundredths*. So also $7 \div 10$, or $7\cdot0 \div 10 = \cdot70$ or $\cdot7$.

In like manner, the 100th part of a number may be represented by shifting the decimal point two places to the left. And, universally, when a whole number, consisting of 1 with ciphers annexed, is used as a divisor, the quotient may be represented by merely shifting the decimal point of the dividend as many places to the left as there are ciphers in the divisor. Thus,

$$860 \div 100 = 860\cdot0 \div 100 = 8\cdot6;$$
$$45 \div 1000 = 45\cdot0 \div 1000 = \cdot045;$$
$$23\cdot47 \div 10000 = \cdot002347.$$

On the same principle, a number is multiplied by 10, or made 10 times as great in value, when its decimal point is transferred one place to the right; by 100, when two places to the right; and so on. Thus,

$$42 \times 10 = 42\cdot0 \times 10 = 420;$$
$$3\cdot0652 \times 100 = 306\cdot52;$$
$$\cdot004 \times 10000 = \cdot00400 \times 10000 = 0040\cdot0 = 40.$$

Suppose, now, we are required to multiply 6·39 by 7·5; the multiplicand is $= \frac{600}{100} + \frac{39}{100}$ or $\frac{639}{100}$; similarly, the multiplier is $= \frac{75}{10}$. Therefore,

$$6\cdot39 \times 7\cdot5 = \frac{639}{100} \times \frac{75}{10} = \frac{639 \times 75}{1000} = \frac{47925}{1000} = 47\cdot925. \textit{ Ans.}$$

Hence, it will be evident that, in decimal multiplication, we may determine the product as if for whole numbers, and then divide it by 1 with as many ciphers annexed as there are decimals in both factors; or, which is the same thing, point off from the right of the product as many decimal places as are contained in both factors.

Examp. Multiply 49·325 by ·0365; also, 4·68 by ·0037.

In the second of these examples, we have made allowance for the figures which are carried from the products of the repeating figure.

49·325	·003777
·0365	4·68
246625	302
295950	2267
147975	15111
1·8003625	·017680

Again, suppose it is required to divide 6·39 by 7·5: $\frac{639}{100} \div \frac{75}{10} = \frac{639}{1000} \div \frac{75}{100} = 63\cdot9 \div 75 = \cdot852$.

Hence, to divide decimally, multiply the divisor by 1 with as many ciphers as will make an integral product; which is just removing the decimal point as many places to the right of the divisor, as there are decimal places in that divisor. Remove the point in the dividend the same number of places to the right. Then divide as in whole numbers, and place a decimal point in the quotient, just when you are proceeding to include the tenths of the dividend in a partial dividend.

Examp. Divide 187·643 by 5·31, and ·015 by ·1847.

531)18764·300 — *Ans.* 35·338

1847)0150·0000 — *Ans.* ·0812

The number of Exercises in Multiplication and Division of Decimals may be extended, if necessary, by prescribing the answers in Section 6th to be multiplied by the given divisors, and the answers in Section 5th to be divided by the given multipliers.

Contracted Multiplication and Division of Decimals.

The following examples of approximation will teach the pupil how to obtain decimal products and quotients with any assigned degree of precision.

Examp. Multiply 2·369578 by 3·58796, preserving accuracy in the product as far as the sixth decimal.

Entire Process.		*Contracted Process.*
2·369578		23′6′9′5′7′8′
3·58796		358796
142	17468	142
2132	6202	2133
16587	046	16587
189566	24	189566
1184789	0	1184789
7108734		7108734
8·501951	08088	8·501951

To understand the contracting operation, observe, that as the number of decimal figures in the two factors is 11, and we wish to retain only 6 in the product, we cut off, by a trait or accent, 5 figures from the right of the multiplicand. We then multiply 23 by the right hand figure of the multiplier, taking in what would have been carried in the complete process; this gives the first partial product. We next mark off 236 to be similarly multiplied by 9, &c.; making all the right hand figures of the several products stand in one column. A right hand figure, besides being increased by carriage, should be further increased by 1, if the figure that should have stood on its right, in the complete process, is greater than, or very nearly equal to, 5.

When the number of figures to be marked off in the multiplicand is greater than the multiplicand contains, supply ciphers on the left, as in the annexed method of multiplying ·684 by ·639, to secure two places of decimals.

0'0'6'84
639
1
2
41
·44

Examp. Divide 46·271 by 6·0579, stopping the quotient at the 6th decimal.

The mode of abbreviation begins by making the divisor a whole number, containing *at least* as many figures as are required in the quotient, the dividend being made conformable, by shifting its decimal point. We then multiply the entire divisor by the first quotient figure, and subtract; then multiply all but the right hand figure of the divisor by the next quotient figure, &c., marking off the figures of the divisor, one by one, from right to left, and neglecting those on the right of the accent, except in the employment of their carriage figures.

6'0'5'7'9'0'0)46271000

Ans. 7·638125

42405300
3865700
3634740
230960
181737
49223
48463
760
606
154
121
33
30

Promiscuous Exercises in Decimals.

1. Add together ·1234, 1·234, and 1234; and from the sum subtract 12·34̇. *Ans.* 1223·01295̇.

2. Multiply ·205 by ·205; and from the product subtract $\frac{1}{3}$ of $\frac{3}{7}$. *Ans.* ·004525.

3. From $\frac{1}{14}$ subtract ·5697. *Ans.* ·0789486.

4. What vulgar fraction is the difference between ·407 and ·407? *Ans.* $\frac{1}{11\,1\,0}$.

5. Divide ·025 by ·000625. *Ans.* 40.

6. Express ·00231 of a foot as the decimal of a mile. *Ans.* ·0000004375.

7. Multiply 27·450867 by itself, employing the contracting process, so as to retain only four decimal figures in the product. *Ans.* 753·5501.

8. Multiply ·52368 by ·06975, retaining six decimal figures. *Ans.* ·036527.

9. Divide, by contraction, 4375 by 64964, retaining six decimal places. *Ans.* ·067345.

10. Divide, by contraction, ·01 cwt. by ·4657328 cwt., retaining five decimals. *Ans.* ·02147.

11. What decimal part of a £. is 16*s.* 11$\frac{1}{2}$*d.* $\frac{1}{4}$*f.*? *Ans.* £·84916.

12. What decimal part of an acre is half a square yard? *Ans.* ·0001033 *ac.*

13. How many French metres, each equal to 39·371 English inches, are there in 30$\frac{1}{2}$ English yards? *Ans.* 27·736.

14. How many imperial gallons, each 277·274 cubic inches, can be contained in a vessel whose capacity is $3\frac{1}{10}$ cubic feet? *Ans.* 19·3195 *gall.*

15. Calculate ·09365 cwt., at 10$\frac{3}{4}$*d.* per lb. *Ans.* 9*s.* 4$\frac{3}{4}$*d.*+

16. Calculate £29. 15*s.* 6$\frac{1}{2}$*d.* at 16·09016*s.* per £. *Ans.* £23. 19*s.* 1·4*d.*

17. Multiply 14·030 by ·01369863. *Ans.* ·182.

18. Divide ·364 by 25·00623. *Ans.* ·014570942.

19. What decimal fraction of 36 da. 3 hrs. 49 sec. is 3 da. 23 hrs. 31 min.? *Ans.* ·1101674.

20. Compute 35·87 cwt. at 3·589*d.* per lb. *Ans.* £60. 2*s.* 0$\frac{3}{4}$$\frac{1}{4}$*d.*

21. If an ounce of gold be worth £3·8, what is the worth of ·04 lb.? *Ans.* £2. 1*s.* 5$\frac{1}{2}$*d.*

22. If ·001 lb. of silver cost ·0738 shilling, what cost 3·28 oz.? *Ans.* 20*s.* $2\frac{3}{11}d.$

23. If $\frac{1}{7}$ of a yard of cloth is worth ·63 shilling, what decimal of a £. is the value of $\frac{9}{11}$ of a yard?

Ans. £·0008.

24. A and B together manufacture 137·6$\frac{1}{2}$ yards of cloth, B's quantity being 9·467 yards less than A's. How much is done by each?

Ans. A. 73·5635 *yds.*

25. What decimal fraction of a lb. is equal to the difference between $\frac{3}{4}$ of a cwt. and ·004 ton?

Ans. ·21 *lb.*

26. The circumference of any circle divided by 3·14159 gives the diameter. Required a multiplier in lieu of that divisor. *Ans.* ·31831.

27. What decimal divisor will reduce years to days, reckoning 365$\frac{1}{4}$ days to a year? *Ans.* ·00273785.

28. Multiply $\frac{1}{8}$ by $\frac{7}{8}$; and let the product contain the same number of figures in its numerator as in its denominator. *Ans.* $\frac{9}{11}$.

29. If, with the view of reducing guineas to crowns, I employ two successive multipliers, the first of them being 50; what must the second be? *Ans.* ·084.

30. If, for the purpose of converting bushels into quarters, I divide by 5$\frac{1}{2}$, what decimal multiplier applied to the quotient will produce the correct result?

Ans. ·6875.

31. If 15·46 times A's money amount to 4·035 times B's, and ·144 of B's money is 23·19 crowns; how much money has each?

Ans. A £10. 10*s.* $1\frac{7}{8}d.$; B £40. 5*s.* $2\frac{1}{2}d.$

32. What number, having ·265 of its 21th part subtracted from it, will become equal to 11 times the difference of 2·9 and ·29? *Ans.* $29\frac{1}{13}$.

33. A. can perform a work in 1·35071, B. in 1·3155, and C. in 1·4616 days. In what time could the three conjointly accomplish it? *Ans.* ·4577358 *da.*

PRACTICE.

In calculating the cost of any considerable number of articles at a given price for each, it is usual to employ a compendious method, which, on account of its convenience and current use in business, has obtained the name of Practice.

Thus the cost of 136 yds. of cambric at 12*s.* 6*d.* the yd. is equal to 136 times 10*s.* + 136 times 2*s.* 6*d.*, *i. e.* 136 *halves* of £1 + 136 *eighths* of £1, or $\frac{1}{2}$ + $\frac{1}{8}$ of £136, or £68 + £17, = £85. The process has consisted in breaking up the given rate into parts which constitute primitive fractions of £1, then calculating for each part, and lastly combining the amounts.

By comparing the methods of Compound Multiplication and Practice, the superior facility of the latter will be apparent.

£0	12*s.*	6*d.*	10*s.* = $\frac{1}{2}$ of each £.	£136.
		12		
			2*s.* 6*d.* = $\frac{1}{8}$	68
7	10	0		17
		11		
				£85.
82	10	0		
2	10	0 = 4 times.		
£85	0	0		

For *Definition*, see Manual.

In commencing instruction in this Rule, such an example as the following may be selected:—

Examp. What cost 173 slates at 3*d.* each?

173*d.*	12)173	3*d.* = $\frac{1}{4}$)173*s.*
3		
	14*s.* 5*d.*	*Ans.* 43*s.* 3*d.*
12)519*d.*	3	
43*s.* 3*d.*	43*s.* 3*d.*	

The teacher may show, first, that the whole amount must be

173 times 3*d.* = 3 times 173*d.*, and that it is expedient then to convert the amount of pence into shillings, &c.; secondly, that 173*d.* may be converted into shillings and odd pence, and then taken 3 times; thirdly, that the method of Practice takes 173 times 3*d.* = ¼ of 173 times 1*s.* = ¼ of 173*s.*

The following Tables will serve for mental instruction and examination by the Teacher. When the table of the aliquots of £1 and 1*s.* is familiar to the Pupil, he may be conducted through the Exercises which we have subjoined to that Table; and it may be expedient that the Exercises in the Manual should be reserved for a promiscuous Praxis after he has wrought all the classified sets which we shall here propose.

Table of Principal Aliquot Parts.

10*s.*	0*d.*	=	£½	2*s.*	6*d.*	=	£⅛	0*s.*	4*d.*	=	⅓*s.*
6	8	=	£⅓	2	0	=	£⅒	0	3	=	¼*s.*
5	0	=	£¼	1	8	=	£1/12	0	2	=	⅙*s.*
4	0	=	£⅕	1	0	=	£1/20	0	1½	=	1/12*s.*
3	4	=	£⅙	0	6	=	1/40*s.*	0	1	=	1/20*s.*

EXERCISES.

1. 263 lbs. of tea, at 5*s.* *Ans.* £65. 15*s.* 0*d.*
2. 365 days, at 1*s.* 8*d.* *Ans.* £30. 8*s.* 4*d.*
3. 1349 yds. of silk, at 2*s.* 6*d.* *Ans.* £168. 12*s.* 6*d.*
4. 153 weeks, at 10*s.* *Ans.* £76. 10*s.* 0*d.*
5. 221 copies of a book, at 6*s.* 8*d.* *Ans.* £73. 13*s.* 4*d.*
6. 1635 steam-boat fares, at 6*d.* *Ans.* £40. 17*s.* 6*d.*
7. 59 oranges at 1½*d.* *Ans.* £0. 7*s.* 4½*d.*
8. 3657 tracts, at 1*d.* *Ans.* £15. 4*s.* 9*d.*
9. 157 lbs. of rice, at 4*d.* *Ans.* £2. 12*s.* 4*d.*
10. The particulars of a school account are, 47 reading-books at 3*s.* 4*d.*, 29 quires of paper at 1*s.*, 35 slates at 3*d.*, and 63 ditto at 2*d.* Required the amount. *Ans.* £10. 4*s.* 11*d.*

When a given price is not an aliquot part of £1 or 1*s.*, it may be resolved into aggregate parts which are aliquots of those integers, and which may be used to calculate aggregate parts of the amount. But it often happens that one portion of a rate is a convenient

aliquot of a previously used portion; and, accordingly, from the amount found by the one may be derived the amount for the other. Thus, the rate 11*s.* 3*d.* may be resolved into 10*s.* and 1*s.* 3*d.*, the former of which is £½, and the latter £$\frac{1}{16}$; but 16 is an inconvenient divisor; and, therefore, when we have found the amount at 10*s.* we should calculate for 1*s.* 3*d.* as the 8th part of 10*s.*, by taking the 8th part of the amount at 10*s.*

Examp. How much money will buy 263 pairs of shoes, at 9*s.* 7*d.* per pair?

The given rate is made up of 5*s.* + 3*s.* 4*d.* + 1*s.* 3*d.*, which are, respectively, £¼, £$\frac{1}{6}$, and ¼ of 5*s.*; or, it consists of 6*s.* 8*d.* + 2*s.* 6*d.* + 5*d.*, which are, respectively, £$\frac{1}{3}$, £$\frac{1}{8}$, and ⅛ of 2*s.* 6*d.* Hence, by setting out with the supposition that the price per pair is £1, which would make the amount £263, we may deduce the answer for the actual rate by either of the following forms:—

5*s.* 0*d.* = ¼	£263	6*s.* 8*d.* = ⅓	£263
3 4 = ⅓	65 15 0	2 6 = ⅓	87 13 4
1 3 = ¼	43 16 8	0 5 = ¼	32 17 6
	16 8 9		5 9 7
Ans.	£126 0 5		£126 0 5

Examp. Calculate 429 yards of calico at 7½*d.*

Here, the given price may be resolved into 6*d.* and 1½*d.*; or it may be derived as ½ of the rate 2*s.* 6*d.*

6*d.* = ½	429*s.*	2*s.* 6*d.* = ⅛	£429
1½*d.* = ¼	214*s.* 6*d.*	0 7½ = ¼	53 12 6
	53 7½		£13. 8*s.* 1½*d.*
	20)268 1½		
Ans.	£13. 8*s.* 1½*d.*		

Let the Pupil be made familiar with the following promiscuous aliquots, and then proceed to the Exercises that are subjoined.

Table of Subordinate Aliquot Parts.

2s. 6d.	=	¼	of	10s.	0d.								
1 8	=	⅛	of	10	0								
1 4	=	1/16	of	4	0								
1 3	=	¼	of	5	0	=	1/16	of	10s.	0d.			
0 10	=	¼	of	3	4	=	⅛	of	5	0	=	1/16	of 10s. 0d.
0 8	=	⅛	of	2	0	=	1/16	of	4	0	=	1/10	of 6 8
0 7½	=	1/32	of	1	3	=	¼	of	2	6	=	⅛	of 5 0
0 6	=	⅛	of	2	6	=	1/16	of	4	0	=	1/10	of 5 0
0 3¾	=	1/32	of	0	7½	=	¼	of	1	3	=	⅛	of 2 6
0 2½	=	⅛	of	1	3	=	⅛	of	1	8	=	1/16	of 2 6
0 1½	=	¼	of	0	6	=	1/10	of	1	3	=	1/20	of 2 6
0 1¼	=	⅛	of	0	7½	=	⅛	of	0	10	=	1/16	of 1 3
0 0¾	=	1/32	of	0	1½	=	¼	of	0	3	=	⅛	of 0 6
ditto	=	1/10	of	0	7½	=	1/20	of	1	3	=	1/40	of 2 6

Exercises.

1. Calculate 4049, at 8½*d.*; at 7½*d.*; at 6¾*d.*
Ans. £143. 8*s.* 0½*d.*; £122. 6*s.* 3½*d.*; £113. 17*s.* 6¾*d.*

2. Calculate 197, at 2*s.* 11*d.*; at 1*s.* 10½*d.*; at 15*s.* 6*d.*
Ans. £28. 14*s.* 7*d.*; £18. 9*s.* 4½*d.*; £152. 13*s.* 6*d.*

3. Calculate 2319, at 5*d.*; at 7*d.*; at ¾*d.*
Ans. £48. 6*s.* 3*d.*; £67. 12*s.* 9*d.*; £7. 4*s.* 11¼*d.*

4. Calculate 2000, at 2*s.* 9¾*d.*; at 4*s.* 2*d.*; at 10*s.* 8*d.*
Ans. £281. 5*s.*; £416. 13*s.* 4*d.*; £1066. 13*s.* 4*d.*

5. Calculate 146, at 14*s.* 8*d.*; at 15*s.* 2½*d.*; at 7*s.* 9½*d.*
Ans. £107. 1*s.* 4*d.*; £110. 17*s.* 4½*d.*; £56. 17*s.* 7*d.*

6. Calculate 365, at 3*s.* 9½*d.*; at 16*s.* 7¾*d.*; at 13*s.* 2½*d.*
Ans. £69. 3*s.* 11½*d.*; £303. 15*s.* 8¾*d.*; £241. 1*s.* 0½*d.*

When the given price falls short of £1, or 1*s.*, &c., by some aliquot part of these integers, it is often expedient to calculate for that aliquot, and subtract; as in the following examples, in which we have deduced the answers from the amounts at £1. and 10*s.*

Examp. 257 pairs of boots at 17*s.* 6*d.* and at 9*s.* 2*d.*

	£.	*s.*	*d.*
2*s.* 6*d.* = $\frac{1}{8}$)257	0	0	
Amt. at 2*s.* 6*d.* = 32	2	6	
Ans.	£224	17	6

	£.	*s.*	*d.*
10*d.* = $\frac{1}{12}$)128	10	0	
Amt. at 10*d.* = 10	14	2	
Ans.	£117	15	10

The following Table may assist in detecting opportunities of such abbreviation :—

Table of Aliquot Complements.

19*s.*	6*d.*	wants $\frac{1}{20}$ of £1.		15*s.*	0*d.*	wants $\frac{1}{4}$ of £1.	
19	4	.... $\frac{1}{30}$		13	4	.... $\frac{1}{5}$	
19	0	.... $\frac{1}{20}$			11	.... $\frac{1}{12}$ of 1*s.*	
18	4	.... $\frac{1}{15}$			10½	.... $\frac{1}{2}$	
18	0	.... $\frac{1}{10}$			10	.... $\frac{1}{6}$	
17	6	.... $\frac{1}{8}$			9	.... $\frac{1}{4}$	
16	8	.... $\frac{1}{6}$			8	.... $\frac{1}{5}$	
16	0	.... $\frac{1}{5}$					

EXERCISES.

1. Calculate 470, at 15*s.*; at 10½*d.*; at 16*s.* 8*d.*
Ans. £352. 10*s.*; £20. 11*s.* 3*d.*; £391. 13*s.* 4*d.*

2. Calculate 532, at 18*s.* 4*d.*; at 10*d.*; at 8*s.* 9*d.*
Ans. £487. 13*s.* 4*d.*; £22. 3*s.* 4*d.*; £232. 15*s.*

3. Calculate 2345, at 19*s.* 8*d.*; at 18*s.* 6*d.*; at 6*s.* 7*d.*
Ans. £2305. 18*s.* 4*d.*; £2169. 2*s.* 6*d.*; £771. 17*s.* 11*d.*

When the given price is an exact number of shillings, it is generally expedient to express it in 10ths of £1, and multiply by it. Thus, 47 yards at 16*s.* = 47 times £·8 = £37·6, or £37. 12*s.* Thus, also, 59 yards at 17*s.* = 59 times £·8½ = £50·1½, or £50. 3*s.*

The Pupil may now proceed to work the Exercises in the Manual, as the illustrations we have yet to give will not become requisite till he has arrived at Ex. 18.

When the given price contains pounds, multiply by the number of pounds, and to the product add the amounts for the smaller denominations, if any, calculated as aliquot parts.

Examp. Find the worth of 573 oz. of gold at £3. 17*s.* 9*d.*; and of 267 cwt. of sugar at £2. 14*s.* 6*d.*

	£573				£267		
	3				2		
15*s.* 0*d.* = ½	1719	0	0	14*s.* 0*d.* = £·7	534		
2 6 = ¼	429	15	0	0 6 = $\frac{1}{40}$	186	18	0
0 3 = $\frac{1}{120}$	71	12	6		6	13	6
	7	3	3				
Ans.	£2227	10	9	*Ans.*	£727	11	6

When the quantity whose value is required contains a fraction, or several denominations, convert it into £. *s.* *d.* as the amount at £1, and then proceed in the usual way. If the fraction or inferior denominations be not convertible into proper money, neglect them till the main integral quantity is computed, and then apply them in aliquot parts to the given rate.

Examp. Calculate 76$\frac{3}{5}$ ells at 13*s.* 6*d.*; and 53 ac. 3 ro. 15 po. at £2. 10*s.* 6*d.* per acre.

	£76. 7*s.* 6*d.*				£53. 16*s.* 10½*d.*		
					2		
10*s.* 0*d.* = ½				10*s.* 0*d.* = ½	107	13	9
3 4 = ⅕	38	3	9	0 6 = $\frac{1}{20}$	26	18	5½
0 2 = $\frac{1}{30}$	12	14	7		1	6	11$\frac{1}{10}$
	0	12	8$\frac{2}{5}$				
Ans.	£51	11	0$\frac{3}{5}$	*Ans.*	£135	19	1$\frac{1}{10}$

Examp. Calculate 373¼ at £12. 15*s.* 9*d.*; and 6 cwt. 1 qr. 17 lb. at 56*s.* 10½*d.* per cwt.

	£373		
	12		
12*s.* 0*d.* = 1/20	4476		
3 0 = ¼	223	16	0
0 9 = 1/80	55	19	0
⅔ = ⅓ of rate	13	19	9
⅓ of do.	4	5	3
.	1	8	5
Ans.	£4775	8	5

	£2. 16*s.* 10½*d.*		
			6
1 qr. = ¼	17	1	3
16 lb. = 1/7	0	14	2½ ½
1 lb. = 1/16	0	8	1½
	0	0	6¾ ⅜
Ans.	£18	4	1⅛ 7/16

When absolute accuracy is not required, the compound quantity, though not entirely convertible into proper money, may be changed into an approximate amount for £1. by a decimal extension of the pence to two or three figures. In converting cwt. qrs. lbs. into their value at £1. per cwt., the qrs. are valued at 5*s.*, and the lbs. at 2½*d.** See the first of the subjoined examples.

The Rule of Practice is applicable to other calculations besides money rates. See the second of the subjoined examples:

Examp. Required the amount of 91 cwt. 3 qrs. 13 lbs. of hops, at £9. 13*s.* 7*d.* per cwt.

Examp. What quantity of grain is reaped from 736 ac. 3 ro. 28 po., the average produce being 4 qrs. 5 bu. 1 pk. per acre?

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* "There is no species of calculation more frequently occurring in counting-houses than weights, nor is there any one more dependent on individual efficiency. There are Tables for interest, ready-reckoners for minor calculations, but only personal attention for cwts., quarters, and pounds; and, therefore, a youth cannot be too well disciplined in exercises so important."—Richson's *Mental Arithmetic (Preface)*.

£91. 17*s*. 3·857*d*.
9

10*s*. 0*d*. = ½	826	15	10·71
3 4 = ¼	45	18	7·93
0 3 = $\frac{1}{20}$	15	6	2·64
	1	2	11·60

Ans. £889 3 8·88 or 9*d*.

736 qrs. 7 bu. 1·6 pks.
4

4 bu. = ½	2947	5	2·4
1 bu. = ¼	368	3	2·8
1 pk. = ¼	92	0	3·7
	23	0	0·9

Ans. Qrs.3431 2 1·8 or 2 pks.

A ready perception of the expedients best adapted to particular quantities and prices can only be attained by practised reflexion.

PER CENTAGES.

In many of the calculations of business and science, the number 100 is employed, as a constant denominator in the expression of rates or ratios. This is to promote facility of computation and comparison.

Thus, if an agent transact business for his employer to the amount of £1575, the remuneration for such agency will be *easily calculated*, if it be fixed at an allowance of so much for every £100 in the amount, say £2 for every £100, or 2 *per cent.* as it is called; then, the agent will receive $\frac{2}{100}$ of £1575 = £15·75 × 2 = £31·5, or £31. 10*s*.

Again, If A invests in trade £250, and gains

£7. 10*s.*, while B invests £40, and gains £1. 8*s.*; then A's rate of gain is $\frac{7\frac{1}{2}}{250}$ of his outlay $= \frac{3}{100}$, or 3 per cent.; B's rate of gain is $\frac{1\frac{3}{4}}{40}$ of his outlay $= \frac{7}{200} = \frac{3\frac{1}{2}}{100}$ or 3½ per cent. So that in the per centage rates, 3 and 3½, we have a *direct comparison* between the gains of A and B.

For *Definition*, &c., see Manual.

The calculation of per centages may be performed either according to the Rule of Proportion, or by using the rate as a fractional multiplier, or by resolving it into aliquot parts of 100; as in the following example:—

Examp. If my gain by selling cloth be £7½ on every £100 which I lay out upon it; what sum do I clear by the sale of a quantity which cost me £260. 12*s.* 6*d.*?

£100 : £260. 12*s.* 6*d.* :: £7½ : £19. 10*s.* 11½*d.*

Ans.

£260. 12*s.* 6*d.* × $\frac{7\frac{1}{2}}{100}$ (*otherwise*) £260. 12*s.* 6*d.*

7½ | 5 p. c. = $\frac{1}{20}$ | 13 0 7½

100)1954 13 9 | 2½ p. c. = ½ | 6 10 3¾

£19 10 11½ £19 10 11½

The above question would be more concisely expressed thus:—Required the gain on £260. 12*s.* 6*d.*, at 7½ per cent.

And it should be observed, that though the rate thus expressed is generally understood to mean £7½ on £100, it may also signify 7½*s.* on 100*s*, or 7½*d.* on 100*d.*, and denotes generally 7½ *hundredths* of the given sum. However, if the assigned rate is in shillings per cent., as 7*s.* 6*d.* per cent., the conventional understanding is 7*s.* 6*d.* per £100, which is £$\frac{3}{8}$ on £100, or generally, $\frac{3}{8}$ per cent.

Perhaps, the best general method for the calculation

of per centages is to express the rate as a fractional multiplier, to simplify the fraction as far as is practicable and expedient, and then to multiply the given sum by the numerator and divide the product by the denominator. Thus, the above example is best suited by taking $\frac{7\frac{1}{2}}{100} = \frac{1\frac{1}{2}}{20}$.

It is often advisable to retain the denominator 100, though the fraction might be reduced to lower terms.

Examp. Required the amount of 6 per cent. on £326. 14*s.* 8½*d.*

Here $\frac{6}{100} = \frac{3}{50}$; but the original fraction is the more convenient. The solution, accordingly, will be performed as in the margin, by multiplying by 6 and dividing by 100, the division being very simply performed by cutting off two right hand figures as the remainder, and taking the figures on the left of it as the quotient.

£326.	14*s.*	8½*d.*	
		6	
£19,60	8	3	
	20		
*s.*12,08			
	12		
d.,99			

Ans. £19. 12*s.* 1*d.*

When a rate is given in *guineas* per cent., £100 is the understood relative integer. Thus, for 3½ guin. per cent. the fractional multiplier would be $\frac{3\frac{1}{2} \times 21}{100 \times 20} = \frac{147}{4000}$. The calculation, however, may proceed as if the guineas were pounds, and then the required correction will be made by adding a 20th part. Or, we may first increase the given sum by its 20th part, and then calculate with the given rate as for £.

The preceding explanations will prepare for the solution of the first 16 Exercises, which are followed by questions of a less elementary character. For the latter kind, the following preparatory exemplification may be requisite:—

Examp. Bought tea at 4*s.* 8*d.* a lb. What is my gain p. c. by selling it at 5*s.* 3*d.*?

Here, 4*s.* 8*d.* gains 7*d.*, or 56*d.* gains $\frac{7}{56}$ of itself, or

£1 on every £8, or £12½ on every £100. But the solution is more concisely exhibited by a Proportion statement, dictated by this form of the question:—If 4*s.* 8*d.* gain 7*d.*, what does 100 gain?

4*s.* 8*d.* : 7*d.* :: 100 : 100 ÷ 8 = 12½ p. c. *Ans.*

Examp. At what rate of shrinkage per cent. will 97 cwt. 3 qrs. 15 lbs. of tobacco lose 3 cwt. 2¾ lbs. by drying?

10963 lbs. : 338·75 lbs. :: 100 : 3·09 p. c. *Ans.*

Examp. What quantity augmented by 3½ p. c. becomes 186?

$\frac{3\frac{1}{2}}{100} = \frac{1}{30}$ of the quantity; $1\frac{1}{30}$ of the quant. = 186.

$1\frac{1}{30} : 1 :: 186 : 30 \times 6 = 180$. *Ans.*

Otherwise:—If 100 becomes 103½, what quantity, at the same rate of increase, becomes 186?

103½ : 100 :: 186 : 180. *Ans.*

Examp. An account is rendered, for present payment of which 4½ p. c. discount is allowed, making the net cash amount £15. 10*s.* 4½*d.* Required the amount rendered.

The allowance is 4½ hundredths of the amount rendered; therefore the cash payment is the remaining 95½ hundredths of the amount rendered, or is to the amount rendered as 95½ to 100; whence,

95½ : 100 :: £15. 10*s.* 4½*d.* : £16. 5*s.* *Ans.*

Examp. If I gain 6 p. c. by selling goods at 12*s.* 6*d.*, what is my gain or loss p. c. by selling them at 9*s.* 6*d.*?

That is, if £100 is made £106 by selling at 12*s.* 6*d.*, what is £100 made by selling at 9*s.* 6*d.*?

12*s.* 6*d.* : 9*s.* 6*d.* :: 106 : 80·56

100 − 80·56 = 19·44 p. c. loss. *Ans.*

Otherwise:—£106 ÷ 12*s.* 6*d.* is the number of yards I bought for £100; therefore this quantity calculated at 9*s.* 6*d.* must be the amount I shall otherwise receive for what cost £100; that is £106 ÷ 12½ × 9½ = £80·56; &c.

Per Centages on Time.

Having, in the preceding article, explained the principles of the calculation of *absolute* per centages, we can easily extend the application of those principles to questions in which the rate per cent. is *affected by the consideration of Time.*

If an account of £52. 10*s.*, due to me at present, remains unpaid for a year, I may charge 5 per cent. for the year's loan of the money, and shall thus receive £2. 12*s.* 6*d.* of remuneration, making the amount £55. 2*s.* 6*d.* If the money is unpaid for two, three, &c. years, I shall charge 10 p. c., 15 p. c., &c. on £52. 10*s.* Uniformly, I shall charge 5 p. c. multiplied by the number of years.

Per Centages on Time, therefore, are at once convertible into absolute per centages. 4 p. c. yearly, for 3 years, is 12 p. c. altogether; 3½ p. c. yearly, for ½ a year, is just 1¾ p. c.

Per Centages on Time belong chiefly to two departments of Arithmetic, called *Interest* and *Discount.*

Simple Interest.

For *Definitions*, &c., see Manual.

When a debt remains unpaid beyond the stipulated term of credit, the creditor cannot, in this country, claim legally a higher rate of interest than 5 per cent. per annum. In former periods of our history the legal rate has been much higher, and it has generally been reduced in proportion as commerce has extended. In Ireland, the United States of America, and some other places, 6 per cent. is the legal interest.

The *market rate* of interest in Britain has been for many years lower than the legal rate.

Examp. Required the interest on £15. 12*s.* 6*d.* for a year, at 5 p. c. per annum.

5 p. c. $= \frac{5}{100} = \frac{1}{20}$ of the principal $=$ 15*s.* 7½*d.* *Ans.*

Examp. Find the interest on £165. 15*s.* for 4 years, at 5 p. c. per ann.

5 p. c. per ann. × 4 yrs. = 20 p. c. = $\frac{20}{100} = \frac{1}{5}$
£165. 15*s.* × $\frac{1}{5}$ = £33. 3*s.* *Ans.*

Examp. What is the amount of £426, for $3\frac{1}{2}$ years, at $3\frac{1}{2}$ p. c. per ann.?

$3\frac{1}{2} \times 3\frac{1}{2} = 13\frac{1}{4}$ p. c., or $13\frac{1}{8}$ *hundredths*;
$4{\cdot}26 \times 13\frac{1}{8} = 4{\cdot}26 \times 105 \div 8 =$ £55. 18*s.* 3*d.* Int.
£426 + Int. = £481. 18*s.* 3*d.* Amount. *Ans.*

Or, the amount might be found thus: The amount of £100 is £$113\frac{1}{8}$; therefore,

100 : $113\frac{1}{8}$ :: £426 : *Ans.*

Although it is expedient that the pupil be well accustomed to regard the product of the rate by the time (in years) as the absolute per cent., it may be pointed out to him, that in many instances the rate and time should be used as two multipliers rather than in composition. Thus, if the rate is $4\frac{1}{2}$ and the time 7 yrs., it is better to multiply by 7 and by $4\frac{1}{2}$ as factors than by $29\frac{3}{4}$, because the multiplier, $29\frac{3}{4}$ hundredths, does not easily admit of simplification. The best general method is to place the rate and the number of years, as factors, over the denominator 100, and then to cancel so far as may be possible and convenient. Thus, 5 yrs. at 6 p. c. = $\frac{5}{1} \times \frac{6}{0} = \frac{3}{10}$ of the Principal; thus, also, $5\frac{1}{2}$ yrs. at $7\frac{1}{2}$ p. c. = 8 × 5, or 40 p. c. = $\frac{4}{10}$ of the Principal. Again, 7 yrs. at 4 p. c. = $\frac{7}{1} \times \frac{4}{0} = 7$ times $\frac{1}{25}$ of the principal.

Months must be reckoned as 12ths of a year, and *Days* as 365ths, because the rate is always *per annum*.

In computing the number of days intervening from one date to another, we reckon the commencing and the concluding day as one, unless it be otherwise specified. Thus, from 12th March to 21st July, we have 19 + 30 + 31 + 30 + 21 = 131 days.

Examp. What is the amount of £257. 10*s.* 6*d.*, lent on 16th June, 1847, and repaid on 2nd March, 1848, with interest at 5 p. c.?

From 16th June to 2nd March, leap year, = 260 days;

5 p. c. for $\frac{111}{365}$ of a year = $\frac{111}{365}$ of $\frac{1}{20}$ of the Principal;
$\frac{1}{20}\frac{111}{365}$ of £257. 10*s*. 6*d* = £9. 3*s*. $5\frac{111}{365}$*d*. Int.

257 10 6 Princ.

Ans. £266 13 $11\frac{111}{365}$ Amt.

When the Principal, the Rate, or the Time, is a required quantity, as in the Exercises following the 24th, employ a Proportion statement, as in the following examples :—

Examp. What principal will gain £17. 17*s*. in 4 years, at $3\frac{1}{2}$ p. c. ?

The interest for one year is $\frac{1}{4}$ of £17·85; therefore, If £100 gain £$3\frac{1}{2}$ in a year, what principal will gain $\frac{1}{4}$ of £17·85 in the same time ?

$3\frac{1}{2}$: $\frac{1}{4}$ of 17·85 :: £100. : $\frac{1}{4}$ of 1785 ÷ 7 = £127·5. *Ans.*

Examp. At what rate p. c. per ann. will £127. 10*s*. gain £17. 17*s*. in 4 years ?

Here, the question is, If £127. 10*s*. gain $\frac{1}{4}$ of £17·85 in a year, what will £100 gain in the same time ?

127·5 : 100 :: $\frac{1}{4}$ of 17·85 : 1785 ÷ 510 = $3\frac{1}{2}$ p. c. *Ans.*

Examp. In what time will £127. 10*s*. gain £17. 17*s*. $3\frac{1}{2}$ p. c. per ann. ?

That is, If $\frac{3.5}{100}$ of the Principal be gained in 1 year, in what time will $\frac{17.85}{127.5}$ of it be gained ?

$\frac{3.5}{100}$: $\frac{17.85}{127.5}$ :: 1 yr. : $\frac{17.85}{127.5}$ × $\frac{100}{3.5}$ = 4 yrs. *Ans.*

Examp. What principal will amount to £145. 7*s*. in 4 years, at $3\frac{1}{2}$ p. c. per ann. ?

The absolute gain on £100 is £14 ; therefore, If £100 amount to £114, what principal will amount to £145. 7*s*. ?

114 : 145·35 :: £100 : £127. 10*s*. *Ans.*

Discount for Time.

For *Definitions*, &c. see Manual.

Bankers and merchants, in lending money, remunerate themselves at the time of lending by deducting from the sum borrowed what should be its interest at the end of the term of credit. It is not, therefore, interest properly so called, which they charge, but something better than interest, which they call Discount. It is, however, calculated in the same way as interest; and, accordingly, the Exercises in Bankers' Discount, in the Manual, may be considered as just a continuation of the Exercises in Simple Interest.

A knowledge of the principle of True Discount is of service for the ascertainment of the real value of many pecuniary speculations.

Suppose I request a person to lend £100 for a year, the use of money being reckoned worth $4\frac{1}{2}$ per cent. per annum. The loan may be transacted in either of two ways, conformably to the true value of credit.

1. If the person charges me £4 *just now*, he now gives me £96, which, if improved by me, at $4\frac{1}{2}$ p. c. per ann., will enable me to pay what I owe at the year's end, viz. £100.

2. If the person charges me £$4\frac{1}{2}$ *at the year's end*, he now gives me £100, which, if improved by me, at $4\frac{1}{2}$ p. c. per ann., will enable me to pay what I owe at the year's end, viz. £104. 3*s*. 4*d*.

It appears, then, that interest at $4\frac{1}{2}$ p. c. payable at a year's end, is equivalent to discount at 4 p. c. payable immediately.

But if, while the worth of money is $4\frac{1}{2}$ p. c. per ann., a person lending me £100 for a year, deducts a present remuneration of £4. 3*s*. 4*d*., he gives me £95. 16*s*. 8*d*., for which I have to pay him £100. at the year's end. So that the interest he has for his money is at the rate of £4. 3*s*. 4*d*. per ann. on £95. 16*s*. 8*d*., or about £4. 6*s*. $11\frac{1}{2}$*d*. p. c. per ann. He charges more than the true discount by the interest of that discount.

Bankers, &c. consider themselves entitled to charge in this

way more than the common rate of interest, on account of the risks to which their capital is exposed.

To find the True Discount that will reduce money, due at some future period, to its present value, is *to* find—not the interest of the whole sum, but—the interest of the present value.

Examp. The sum of £345. is payable 3 years hence. What discount should be given as a compensation for paying it at present; the use of money being reckoned worth 5 p. c. per ann.?

Here, the Present Value, were it improved at 5 p. c. per ann. for 3 yrs., should amount to £345.; and, as 5 p. c. per ann. for three years is 15 p. c., we inquire, what sum increased by $\frac{15}{100}$ or $\frac{3}{20}$ of itself amounts *to* £345. Accordingly, $345 \div 1\frac{15}{100} = 6900 \div 23 =$ £300, the Pres. Val., or £45, the Disc. *Ans.*

The usual mode of solution, however, is by a Proportion statement, employing £100 as a given present value. Thus, in the question just resolved, there *is* given £100 as the present worth of £115; that is, £100 paid at present = £115 to be paid 3 yrs. hence. Accordingly, we say, If £100 is the present value of £115, what is the corresponding present value of £345?

115 : 100 :: £345 : £300 Present Worth, £345 Future Worth,

Ans. £45 Discount.

Or, we may find the Discount by the direct inquiry, If £15 is the discount on £115, what is the corresponding discount on £345?

115 : 15 :: £345 : £45. *Ans.*

It is occasionally convenient to consider that the true discount of any sum which is due at a future period is that fraction of the sum which has the absolute rate per cent. for a numerator, and 100 + that rate for a denominator; and that the true present worth of any sum is that fraction of it which has 100 for a numerator, and 100 + the absolute rate for a denominator. Thus, in the above example, the discount is $\frac{15}{115} = \frac{3}{23}$ of £345; and the present worth is $\frac{100}{115}$ or $\frac{20}{23}$ of £345.

The two following examples will serve to illustrate the more difficult kinds of calculation that occur in the Rule of Discount.

Examp. If I purchase dresses at 10*s.* 3*d.*, with 6 months credit, and sell them at 12*s.* 9*d.*, with 3 months credit; what is my immediate gain p. c.?

Here, as no rate per cent. is mentioned, legal interest is to be understood. Accordingly, the use of money for 6 months is worth $2\frac{1}{2}$ p. c., and for 3 months it is $1\frac{1}{4}$ p. c. The present worths, therefore, of 10*s.* 3*d.* and 12*s.* 9*d.* are obtained by the following proportions:

$101\frac{1}{4}$: 100 :: 12*s.* 9*d.* : 12*s.* $7\frac{1}{2}$*d.* P. V. of 12*s.* 9*d.*
$102\frac{1}{2}$: 100 :: 10*s.* 3*d.* : 10*s.* 0*d.* P. V. of 10*s.* 3*d.*

The article costs me 10*s.* cash, and brings me 12*s.* $7\frac{1}{2}$*d.* cash. On 10*s.*, therefore, my gain is 2*s.* $7\frac{1}{2}$*d.*; and, to find my gain p. c.,

10*s.* : 2*s.* $7\frac{1}{2}$*d.* :: 100 : $25\frac{14}{41}$ p. c. *Ans.*

Examp. What should I charge, per yard, for cloth which stands me 19*s.* per English ell, that I may now have 25 p. c. profit, after allowing a discount of 5 p. c. and 6 months credit?

19*s.* an ell, with 25 p. c. profit, is 19*s.* a yard, which is to be the present worth of my receipts. Then, as 6 months credit is worth $2\frac{1}{2}$ p. c.,

100 : $102\frac{1}{2}$ :: 19*s.* : 19·475*s.* a yd., payable in 6 mo.
95 : 100 :: 19·475*s.* : 20*s.* 6*d.* a yd. must be charged, to allow 5 p. c. discount. *Ans.*

Division into Proportional Parts.

When several numbers have each the same ratio to several other numbers, the ratio subsists also in the sum or difference of any corresponding terms in the two series.

Thus, let there be three numbers, 6, 9, and 4, and three other numbers, 36, 54, and 24, the latter being, respectively, 6 times the former; then, the

6 : 36	19 : 114
9 : 54	15 : 90
4 : 24	5 : 30
	7 : 42

sum of the three is, in the one case, 19, and in the other 6 times 19; the sum of the two first is 15 and 6 times 15; the difference of the second and third is 5 and 6 times 5; the sum of the second and third diminished by the first is 7 and 6 times 7; &c.

This persistency of the ratio is easily accounted for. In comparing the sum of *once* 6 and *once* 9, with the sum of *six times* 6 and *six times* 9, we must expect the sum of six times each of the numbers, to be six times the sum of once each of the numbers; and so of the other combinations, &c.

Now a recognition of this persistency enables us easily to resolve a given amount into as many aggregate parts, having the same interproportion, as any given numbers. We are also enabled hereby to determine numbers whose difference and ratio are given.

For *Definition*, see Manual.

Examp. Divide £98. 2*s.* 2*d.* into four parts, proportioned as the numbers 12, 15, 16, and 18.

The first divided by 12, the second by 15, &c., are to yield the same quotient. Now, as the sum of all the required parts is given, we have to compare it with the sum of all the proportional parts, to determine the general ratio.

61 : 12 ::	£98. 2*s.* 2*d.*	:	£19	6*s.*	0*d.*	
— : 15 ::	———	:	24	2	6	*Ans.*
— : 16 ::	———	:	25	14	8	
— : 18 ::	———	:	28	19	0	

Examp. Divide £98. 2*s.* 2*d.* into four parts, which multiplied, respectively, by 12, 15, 16, and 18, shall give equal products.

The first divided by $\frac{1}{12}$, the second by $\frac{1}{15}$, &c., are to yield the same quotient. Therefore, the four parts are to be proportioned as $\frac{1}{12}$, $\frac{1}{15}$, $\frac{1}{16}$, and $\frac{1}{18}$; or, multiplying these fractions by their least common denominator, 720, which will not alter their proportional relation, we obtain 60, 48, 45, and 40, correspondent to

which the answers will be found, £30. 10*s.*, £24. 8*s.*, £22. 17*s.* 6*d.*, and £20. 6*s.* 8*d.*

Examp. Divide 84 guineas into three parts, so that for every 5 guineas in the first there may be 6 in the second, and for every 4 guineas in the second there may be 9 in the third.

The second is $\frac{6}{5}$ of the first, and the third is $\frac{9}{4}$ of the second, or $\frac{9}{4}$ of $\frac{6}{5}$ of the first, *i. e.* $\frac{27}{10}$ of the first. So that the three parts are as 1, $\frac{6}{5}$, and $\frac{27}{10}$, or as 10, 12, and 27. Dividing 84 guin. in the proportion of these numbers gives $17\frac{1}{7}$, $20\frac{4}{7}$, and $46\frac{2}{7}$, guin. *Ans.*

Examp. Three persons, A, B, and C, are to have a certain sum divided among them, in such a manner that $\frac{1}{4}$ of B's share shall be equal to $\frac{1}{2}$ of A's, and $\frac{1}{3}$ of A's to $\frac{1}{4}$ the sum of B's and C's; by this means the difference of the shares of B and C will be 10*s.* 6*d.* Required the whole sum, and each share.

B : A :: 5 : 8; therefore, A gets $\frac{5}{8}$ of B;

A : B + C :: 5 : 8; therefore, B and C together get $\frac{5}{8}$ of A, *i. e.* $\frac{25}{32}$ of B, or C gets $\frac{25}{57}$ of B. Hence, the shares of A, B, and C, are as $\frac{5}{8}$, 1, and $\frac{25}{57}$, or as 40, 25, and 39.

Now B's and C's shares differing by 10*s.* 6*d.*, and their proportional parts by 39 — 25, or 14, we have,

14	: 40	:: 10*s.* 6*d.*	:	30*s.*	0*d.*	to A.	} *Ans.*
—	: 25	:: ———	:	18	9	to B.	
—	: 39	:: ———	:	29	3	to C.	
—	:104	:: ———	:	78*s.*	0*d.*		

Sometimes the proportional parts require to be determined by compounding ratios, as in the following example:—

Examp. A merchant bequeaths £1000 among six clerks, in proportion to their salaries, and the periods they have held their situations. Now, one of them has held his situation 5 years, and his salary is £120; two

of them 4 years, with salaries of £75 each; and the rest 2 years, with salaries of £60 each. Required their several shares.

120 × 5 = 600 the proportional part for one;
150 × 4 = 600 ditto for two others;
180 × 2 = 360 ditto for three remaining.

Accordingly £1000 divided in the proportion of these numbers, gives £384$\frac{8}{19}$ to one, £192$\frac{4}{19}$ to each of two, and £76$\frac{12}{19}$ to each of three.

FELLOWSHIP.

For *Definition*, see Manual.

The Rule of Fellowship is so direct an application of the Rule of Proportional Parts, that the Exercises under the former title may be regarded as a continuation of those under the latter.

We shall here, however, give some illustration of Compound Fellowship, or Fellowship with Time.

Three things are principally to be considered in Compound Fellowship,—the capital formed by the stocks of the several partners, the respective periods of the continuance of these stocks in trade, and the respective gains or losses on the stocks.

The relations of these three particulars are the following:—

The Gains (or the Losses) are as the products of Stock and Time. The Stocks are as the quotients of Gain by Time. The Times are as the quotients of Gain by Stock.

Examp. A, B, and C, are mercantile partners: A's stock being £540, which he continues for 5 months; B's £460, for 3 months; C's £560, for 6 months. They gain by the entire capital, £248. Required each partner's share of this gain.

Here, £540 gains in 5 months what £540 × 5 would gain in 1 month; also £460 for 3 months = £460 × 3 for 1 month; and £560 for 6 months =

£560 × 6 for 1 month. The gains of A, B, and C, therefore, are as 2700, 1380, and 3360.

7440	2700	£248	£90,	A's gain.	*Ans.*
——	1380	——	46,	B's do.	
——	3360	——	112,	C's do.	
	£7440		£248		

Examp. Suppose the data to be, each of the above gains, and each of the times, together with the difference of the stocks of A and B = £80; to find each stock.

A's gain per month is £90 ÷ 5, B's £46 ÷ 3, C's £112 ÷ 6; therefore, their stocks are as 18, $15\frac{1}{3}$, and $18\frac{2}{3}$, or as 54, 46, and 56.

54 — 46	54	£80	£540,	A's stock.	*Ans.*
——	46	—	460,	B's do.	
——	56	—	560,	C's do.	

Examp. Suppose, now, the data to be, each of the gains, and each of the stocks, together with the sum of the times = 14 months; to find the time of each investment.

A gains $\frac{9}{14}$ of his stock, B $\frac{4}{14}$, and C $\frac{1}{14}$; their times must, therefore, be proportioned as $\frac{1}{9}$, $\frac{1}{4}$, and $\frac{1}{1}$; and if 14 months be divided according to these proportional parts, we obtain A's time 5, B's 3, C's 6, months. *Ans.*

SQUARE AND CUBIC MEASURE.

A Square is a surface contained by four sides, equal to each other, and joined perpendicularly.

Square Measure is so called from its representing the number of times that a certain square unit is contained in a given surface or area.

The exercises in Square Measure, in the Manual, have reference only to quadrilateral surfaces, with perpendicular sides, as the figure A B F E in page 42nd.

And of such figures it is easily demonstrable, that the number of lineal units in any one of the sides multiplied by the number of lineal units in a side that is perpendicular to that one, yields the number of square units in the whole surface.

Suppose the length is 8 inches, and the breadth 3 inches, then, 8 × 3, or 24 square inches, is the surface measure of the figure.*

For *Definition* of Square Measure, see Manual.

A Cube is a solid contained by six superficial sides, equal to each other, and joined perpendicularly.

Cubic Measure is so called from its representing the number of times that a certain cubical unit is contained in a given solid or volume.

As the product of the number of units in length by the number of units in breadth, is the number of square units in one superficial side, so it will represent also the number of cubical units for 1 unit of depth; and, therefore, the product of the units of length and breadth, multiplied by those of depth or thickness, will give solid measure.

For *Definition* of Cubic Measure, see Manual.

The Exercises in Square and Cubic Measure are so elementary as to render an exhibition of examples here unnecessary. Only, it must be remembered, that in performing these Exercises, the dimensions must be applied to each other under a uniform denomination. Thus, 3 ft. 3 in. × 2 ft. 11 in. = $3\frac{1}{4}$ ft. × $2\frac{11}{12}$ ft. = (13 × 35) ÷ 48 = 9 sq. ft. 69 sq. in.

*** In the supplementary part of this volume we have treated of the SQUARE ROOT and CUBE ROOT; and from the illustrations there furnished, the teacher can select as much as he considers necessary to prepare the pupil for the evolutions proposed in the Manual.

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* See Note on *Geometrical Multiplication*, p. 11

SUPPLEMENTARY RULES.

SUPPLEMENTARY RULES.

Interest or Discount on Bills.

A Bill, or Bill of Exchange, is a written and recognised claim, that a debt shall be paid at a certain time. It is written on a stamped paper, the cost of which is regulated by a Stamp Table.

A Bill is first subscribed by the party to whom the money is due, and presented to the party by whom it is to be paid. The latter, in acknowledging the claim, writes his name at the bottom or across the middle of the Bill, with the word *Accepted*, and returns it to the former party. This kind of document is therefore termed an *Acceptance*.

A written engagement, of similar import, is often expressed in the form of a promise, subscribed only by the party who owes the money. This kind of document is therefore termed a *Promissory Note*.

Form of an Acceptance.

£156. London, May 26th, 1847.

Three Months after date, pay to my order, One hundred and Fifty-six Pounds, value received.

James Clarke.

To Mr. William Emerson,
Merchant, Liverpool.

Accepted, William Emerson.

Form of a Promissory Note.

£156. London, May 26th, 1847.

Three Months after date, I promise to pay to Mr. James Clarke, or order, One hundred and Fifty-six Pounds, value received.

William Emerson.

With respect to the first of these forms, Clarke is said to *draw* the Bill on Emerson at 3 months; and as *culendar* months are always understood in such transactions, the term of the Bill expires on 26th August. No Bill, however, is presented for payment till the *third day* after the period specified, there being always an allowance of three days of *grace*, as they are called.

When an acceptance or a promissory note is drawn out as above, it is said to be *negotiable;* that is, it may be transferred by Clarke as a payment to another person; in which case, Clarke, when parting with it, writes his own name on the back of it. This is called *Indorsing* the Bill.

The person who holds the Bill at the time it is due presents it to Emerson for payment, and, should Emerson fail to pay it, may have recourse to Clarke; but the acceptor is liable for all legal expenses incurred on a Bill which has not been duly *honoured* or paid.

Now, while a Bill is a convenient means of providing for the punctual discharge of a pecuniary claim, and for the transmission of a payment to a distance, the drawer or the holder is deprived of the use of the money for the time the Bill has to run. It is of value to him only as a form of seeurity, till the money becomes due; unless he can get some one to cash it before the expiration of the term; and, in this case, the charge called *Discount* is made by the party who gives ready

money in exchange for the Bill, and the ready money by which the Bill is cashed is called the *Proceeds*.

Examp. Suppose a bill for £420 is dated 15th March, at 4 months, and that the holder of it gets it discounted at legal interest on 25th April; what will he then receive for it?

The term of 4 months expires on 15th July, and the bill is payable on 18th July. It is cashed, however, on 25th April, which is 84 days before due; and therefore the sum obtained will be £420 diminished by 84 days' interest on £420. 0*s.* 0*d.*

$\frac{420 \times 5 \times 84}{365 \times 100} = \frac{21 \times 84}{365} =$ £ 4. 16*s.* 8*d.* Disc.

Ans. The proceeds are £415. 3*s.* 4*d.*

NOTE:—Bankers seldom discount bills which have more than 3 months to run; and as interest and true discount differ by a very small sum when the period is short, and the former is much more easily calculated than the latter, the commercial propriety of the principle of Bankers' Discount needs not be disputed.

EXERCISES.

1. M. N. grants a Promissory Note for £25. 7*s.* 6*d.*,* dated 12th June, at 2 months. It comes into A. B.'s possession on 28th of June, who then gets it discounted at 4½ per cent. How much does A. B. receive for it?

Ans. £25. 4*s.* 6*d.*

2. T. W. grants an Acceptance for £227. 1*s.* 8*d.*, dated 14th May, at 4 months. The bill is discounted on 12th July, at 4 p. c. Required the discount.

Ans. £1. 13*s.* 4*d.*

3. Required the proceeds of a bill per £326. 15*s.*, dated 29th Aug., at 3 months, and discounted 19th Sept., at 5 p. c. *Ans.* £323. 8*s.* 8*d.*

* In computing interest for days, or discount on bills, mercantile men omit the shillings from the sum if under 10*s.*, and reckon 10*s.*, or shillings above 10, as £1. Fractions of 1*d.* in the interest are reckoned as 1*d.* if equal to or greater than ½*d.*; otherwise they are rejected.

4. A banker gives cash in one day for six bills, of £25, £50, £75, £80, £85, and £90, respectively, at 3 months. What is his present gain by deducting legal interest instead of true discount? *Ans.* 1*s.* 3*d.*

5. A person procures the use of £100 for a year, by getting a 3 months bill discounted at legal interest, for that sum, at the beginning of every quarter, paying 4*s.* 6*d.* for each stamp. How much will the loan have cost him at the year's end? *Ans.* £6. 1*s.* 8½*d.*

6. A bill per £85, dated 13th February, at 3 months, was discounted 1st April; and the banker, besides charging legal interest, retained ½ per cent. commission. How much did the holder of the bill receive, and at what rate per cent. per ann. did the banker lend his money? *Ans.* £84. 1*s.*; 9·1678 p. c.

TARE AND TRET.

On many articles in wholesale trade it is customary to allow certain deductions of weight, on account of packages, &c. The chief of these allowances are called DRAFT, TARE, and TRET.

The entire weight of any commodity, including box, packing material, &c., is called the *Gross Weight.*

Draft is an allowance per cask, bag, &c., on the gross weight, that the weight may hold out in retail selling. When this allowance is made, it must be deducted before Tare.

Tare is an allowance for the weight of the cask, or other article containing a commodity. *Real* Tare is the actual amount of such weight; *Customary* Tare is an established allowance for it; *Proportionate* Tare is an allowance for it by a rate per cwt. or per cent.; *Average* Tare is an allowance of so many lbs. per package, being the ascertained average weight of a few packages.

Tret is an allowance on account of liability to waste,

and is now confined to a few articles, as Pimento, Roman Vitriol, &c. This allowance is 4 lbs. on 104 lbs., or 1-26th of the remainder after deduction of all other allowances.

The remainder after deduction of Tare, when any further allowance is made, is called *Suttle;* and the remainder after all deductions is called *Net* Weight.

Examp. Required the net weight of 3 puncheons of molasses, weighing gross 35 cwt. 1 qr. 21 lbs.; tare, 3 qrs. 12 lbs. per puncheon.

Gross weight	35 cwt.	1 qrs.	21 lbs.
Tare	2	2	8
Net weight	32 cwt.	3 qrs.	13 lbs. *Ans.*

Examp. What is the net weight of 49 bags of pimento, weighing gross 39 cwt. 2 qrs. 25 lbs.; draft 1 lb. per bag; tare 4 lbs. per cwt.; and tret 4 lbs. per 104 lbs.?

Gross weight	39 cwt	2 qrs.	25 lbs.
Draft	0	1	21
	39	1	4
Tare $\frac{1}{28}$	1	1	$17\frac{1}{7}$
	37	3	$14\frac{6}{7}$
Tret $\frac{1}{26}$	1	1	$23\frac{17}{26}$
Net weight	36 cwt.	1 qr.	$19\frac{9}{13}$ lbs. *Ans.*
or	36	1	20*

EXERCISES.

1. Find the net weight on 2 hhds. of sugar, weighing gross 16 cwt. 2 qrs. 10 lbs., and 17 cwt. 2 qrs. 20 lbs., respectively; tare on the former, 1 cwt. 1 qr. 20 lbs.,

* Fractions under $\frac{1}{2}$ lb. are neglected in commercial allowances; and if equal to or greater than $\frac{1}{2}$ lb. they are reckoned as 1 lb. The Exercises, however, have their exact answers annexed.

and on the latter 1 cwt. 2 qrs. 15 lbs. Find also the value, at 54*s.* per cwt. net.

Ans. 31 cwt. 23 lbs.; £84. 5*s.* $1\frac{1}{17}d.$

2. At $3\frac{1}{4}d.$ per lb. net, what will be the cost of 7 hhds. of tobacco, averaging 14 cwt. 1 qr. 12 lbs.; allowance for draft and shrinkage 17 lbs. per hhd.?

Ans. £150. 16*s.* $3\frac{1}{4}d.$

3. On 3 chests of tea, each 107 lbs., allowance is made of draft 1 lb. per chest, and tare 23 lbs. per ditto; what is the value at 3*s.* 7*d.* per lb. net?

Ans. £44. 12*s.* 3*d.*

4. What is the net weight of 80 chests of cassia lignea; gross weight 54 cwt. 3 qrs.; draft 1 lb. per chest; tare 25 lbs. per cwt.?

Ans. 41 cwt. 3 qrs. $25\frac{8}{28}$ lbs.

5. Required the net weight of 10 hhds. of tobacco, weighing 107 cwt. 3 qrs. 22 lbs., allowing for shrinkage 3 per cent., and for tret 4 lbs. per 104 lbs.

Ans. 100 cwt. 2 qrs. $20\frac{1}{4}$ lbs.

6. Required the net weight of 63 bags of pimento; gross weight 56 cwt. 1 qr. 3 lbs., draft 1 lb. per bag, tare 4 lbs. per cwt., tret as usual.

Ans. 51 cwt. 2 qrs. $17\frac{5}{7}$ lbs.

7. On 50 casks of olive oil, weighing 483 cwt. 3 qrs. 19 lbs., the draft is 4 lbs. per cask, tare 18 lbs. per cwt. Required the net measure estimated at 9 lbs. per gallon, and also the cost at £52 per tun.

Ans. 19 tuns 3 hhds. $58\frac{1}{10}\frac{1}{4}$ gall.; £1039. 1*s.* $11\frac{1}{17}\frac{8}{28}\frac{1}{28}d.$

8. The net weight of a quantity of goods is 33 cwt. $22\frac{3}{4}$ lbs.; the deductions having been, for tare 14 lbs. per cwt., and for tret 4 lbs. per 104 lbs. Required the gross weight. *Ans.* 39 cwt. 1 qr. 24 lbs.

9. Tare and the usual tret being allowed on a quantity of goods weighing 17 cwt. gross, the amount of the tare is found to be exactly double that of the tret. Required the net weight. *Ans.* 15 cwt. 20 lbs.

EQUATION OF PAYMENTS.

EQUATION OF PAYMENTS is the reduction of unequal terms of credit to equal. It is a Rule whose object is to find, at what time the discharge of a whole debt, by one payment, will be equivalent to its discharge by partial payments at different periods.

Examp. If A. owes B. £240, payable in $1\frac{1}{2}$ year, and £332, payable in $3\frac{1}{4}$ years; in what time will a single payment of £572 discharge both debts?

Dealing with this question upon the principle of Bankers' Discount, or Interest, as is always done in business, we find the

Proportionate Int.	on	£240	=	240	×	$1\frac{1}{2}$	= 360
Ditto	on	332	=	332	×	$3\frac{1}{4}$	= 1079
Ditto	on	572	=	572	×	(?)	= 1439

We have taken proportionate interests, each 20 times the real interest, to save the needless trouble of taking $\frac{1}{20}$, or 5 p. c.; and hence $1439 \div 572 = 2 \cdot 515734$ yrs., the equated time for paying both debts at once.

According to this mode of equating the time, A deprives B of the use of £240 for 1·015734 year, which would yield £12·1888 interest; but, in compensation, A allows B the use of £332 for ·734266 year, which would yield the same sum of interest, or bankers' discount. This is, however, more than a just compensation. B's *loss* of the interest of £240, for the period by which the equated time exceeds $1\frac{1}{2}$ year, should be compensated by his *gain* of the *true* discount of £332 for the period by which the equated time precedes $3\frac{1}{4}$ years; and, therefore, the equated time ought to be investigated in such a way as will make the interest which B loses equal the discount which he gains.

This correction was first advocated by Malcolm, of Aberdeen, in his excellent System of Arithmetic (1730). But the solution of problems according to his theory is very tedious, and requires an algebraical evaluation by the method of Quadratics. At the conclusion of his illustrations he admits that for the purposes of business the common Rule is accurate enough, and expresses his apprehension that he will be considered by some to have

bestowed too great pains on a subject of little practical importance.

No arithmetical topic has been more keenly debated than Equation of Payments; but it is now generally admitted that, for two payments, Malcolm's rule is correct, while, for three or more payments (which requires first to equate two payments, then to equate their sum for the time found with another payment, &c.), it is not only intolerably tedious, but assailable by the consideration that compound interest should then be allowed.*

By a quadratic equation we find the true equated time for the above example to be $2\frac{1}{2}$ years; in proof of which the interest of £240 for 1 year, *forfeited* at the end of $2\frac{1}{2}$ years, is balanced by the true discount of £332 for $\frac{1}{2}$ of a year *gained* at the end of $2\frac{1}{2}$ years; the interest and the discount being each £12. A, in effect, pays B £240 *plus* int. for 1 year beyond the due term, i.e. £252, and pays him also £332 *minus* disc. for $\frac{1}{2}$ of a year before the due term, i.e. £320; and thus B receives £572, the exact sum of the debts.

But another theory of Equation of Payments, commonly called Kersey's Rule, deserves mention, viz. that which estimates the equated time by the improvement of which the present worths of the debts are susceptible. Thus, the present worth of £240 payable in $1\frac{1}{2}$ year is £223·2558, and that of £332 for $3\frac{1}{2}$ years is £285·5914; therefore, £508·8472, paid at present, would discharge both debts; and as that sum would amount to £572, at legal interest, in 2·4822 years, that period is an equation of the given times.

This method, which certainly presents an appearance of equity, has the disadvantage of varying the equated time as the time of the first payment approaches. For example, in half-a-year after the above debts are constituted, the problem stands thus:—£240 is due in 1 year, and £332 in $2\frac{1}{2}$ years; required the equated time. By the method of reducing both debts to their present worths, the answer will now be 1·9814 year beyond the half-year, or 2·4814 years from the original date; which does not agree with the former answer. Malcolm's Rule involves no such inconsistency.†

* See Keith's *Arithmetician*, Art. *Equation of Payments at Comp. Int.*, for a universal Rule deducible alike from Kersey's and from Malcolm's principle. See also Keith's Demonstrations of the Rule in the *Key* to his *Arithm.*

† The best mode of applying Malcolm's Rule is to equate from

In all but the two last of the subjoined Exercises, the answers are adapted to the common rule, which is exemplified at the beginning of this article, and which may be stated in general terms, as follows:—

Multiply the given debts by their respective times, expressed in one denomination, and divide the sum of the products by the sum of the debts.

The two last Exercises will, to those who are acquainted with algebra, afford opportunity of comparing the results of the three methods.

EXERCISES.

1. If £100 is payable in $2\frac{1}{2}$ years, and £150 in $3\frac{1}{2}$ years; at what mean term will the payments become due together? *Ans.* $3\frac{1}{10}$ mo.*

2. Find the mean term for payment of £125, £256, and £260, due in 4, 3, and 6 months, respectively. *Ans.* 4·412 mo.

3. I owe £52. 7*s*. 6*d*., payable at the end of $4\frac{1}{2}$ months, £80. 10*s*. at $3\frac{1}{2}$ months, and £76. 2*s*. 6*d*. at 5 months. Equate the times for one payment. *Ans.* 4·297 mo.

4. I owe £237. 12*s*. 6*d*. payable as follows: $\frac{1}{4}$ of it in $1\frac{1}{4}$ year, $\frac{1}{3}$ of the remainder in $1\frac{1}{4}$ year, and the rest in $2\frac{1}{4}$ years. Required the equated time for paying the whole. *Ans.* 1 yr. 8 mo.

5. On April 12th £150 are due, on April 28th £270. 10*s*., and on June 7th £360; what is the

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the end of the first term; thus £240 is due at a certain time, and £332 is due $1\frac{1}{4}$ year after that time; find the equated period.

If we put x for the equated time, the int. of £240 for x yrs. must equal the disc. of £332 for $(1\frac{1}{4} - x)$ yrs.

$$\text{Or, } 12x = \frac{332 \times 5\ (1\frac{1}{4} - x)}{100 + 5\ (1\frac{1}{4} - x)}$$

Whence, $145{\cdot}25 = 148{\cdot}25\ x - 3\ x^2$, being resolved, gives $x = 1$ year after the time at which £240 is due; which is the same as $2\frac{1}{2}$ years formerly shown to be the answer according to Malcolm's Rule.

* Fractions of a month may be converted into days, at 30 days to a month.

average time of payment, if the parties agree that the three payments shall be discharged at once?

Ans. May 13th.

6. Suppose £168 are to be paid at the end of 7 months, and £113. 8*s.* at the end of 12 months; required the time for paying the whole by one payment, according to the common Rule, and also by the methods of Kersey and Malcolm.

Ans. C. 9·015, K. 8·99, M. 9, months.

7. If £152 are due 7 months hence, and £166 are due 9 months hence; equate these times for one payment, by Kersey's, Malcolm's, and the common Rules.

Ans. C. 8·044, K. 8·04, M. 8·02, months.

MARINE INSURANCE.

A MARINE INSURANCE is a contract insuring an assigned value of property risked at sea.

The document containing the terms of insurance is called a *Policy;* the individuals undertaking separate portions of the risk are called *Insurers* or *Underwriters,* and the remuneration they exact is the *Premium* of Insurance.

The Insurance *Broker* is the agent who extends the Policy, and procures the signatures of the Underwriters; and they pay him 1*s.* for every guinea or pound of premium, according as the rate is expressed in guineas, or in pounds or shillings. When merchants act as agents in effecting insurances, they usually charge the insured ½ per cent. commission, and sometimes engage, for an additional ½, or more, per cent., called *del credere,* to guarantee the solvency of the Underwriters.

The Policy Stamp Duties on Sea Insurances are, for a premium not exceeding 10*s.* per cent., 3*d.* on every £100 insured; not exceeding 20*s.*, 6*d.*; not exceeding 30*s.*, 1*s.*; not exceeding 40*s.*, 2*s.*; not exceeding 50*s.*, 3*s.*; for any premium exceeding 50*s.*, 4*s.* Policy Duty is charged on any part of £100 as if it were £100; thus, the duty on £320 would be rated on £400.

In time of war or prevailing piracy a high premium is often exacted, with a stipulation to return part of it if the vessel sail with convoy of a ship of war, and arrive safe at her port.

Sometimes the value assigned for insurance is so much greater

than the value of the property at risk, that, in case of a total loss, the sum obtained from the insurers will cover the expense of the insurance.

Examp. Required the cost of insuring £840, on a cargo from Leith to London, at 15*s.* per cent., policy duty 6*d.* per cent., commission ½ per cent.

Prem. on £840	= 15*s.* × 8·4...	£6	6	0
Pol. on £900	= 6*d.* × 9 ...	0	4	6
Com. on £840	= £840 ÷ 200...	4	4	0

Ans. £10. 14*s.* 6*d.*

Examp. Insured £1250 on vessel and freight, at 2¼ guineas per cent., commission ½ p. c., policy 3*s.* p. c. Find the whole expense.

Prem. if at £2¼ p. c.	= £2¼ × 12½...	£28	2	6
Ditto at 2¼*s.*	= 1 *twentieth* ...	1	8	1½

Prem. at 2¼ guineas p. c.	£29	10	7½
Com. at 10*s.* p. c.	6	5	0
Pol. 3*s.* p. c. on £1300	1	19	0

Ans. £37. 14*s.* 7½*d.*

Examp. What was the cost of insuring £4500, on a cargo from Bahia to Hamburgh, at 5 guineas per cent., to return £2 per cent. for convoy and arrival, which took place; policy 4*s.*, commission ½, del credere ½, per cent.?

Prem. on £4500 at £3¼ p. c.	£146.	5*s.*
Pol., Com., and Guar., 24*s.* p. c.	54	0

Ans. £200. 5*s.*

Examp. What amount must be insured to cover £626. 12*s.* 6*d.*, prem. 2½ guin., pol. 4*s.*, agent's com. ½, per cent.?

Prem., Pol., and Com. = £3·325 p. c. Now an insurance for £100 at that rate would, in case of a

total loss of £96·675, secure a recovery of the latter sum together with the insurance costs; therefore,

96·675 : 100 :: £626·625 : £648. 3*s*. 6½*d*. *Ans*.

In this calculation a small excess of policy duty on the deficiency from exact hundreds has been neglected. We might have included it in the covered interest, by adding the amount of duty on £700, viz., 28*s*. to £626·625, and then finding what will cover £628·025 with the given premium and commission; thus—

96·875 : 100 : : £628·025 : £648. 5*s*. 8*d*. the full amount of covered interest; for, by insuring to this amount, the insured party, in case of a total loss of £626. 12*s*. 6*d*., will recover as follows:—

Value at risk	£626	12	6
Prem. and Com. on £648. 5*s*. 8*d*. at £3½ p. c.	20	5	2
Policy 4*s*. p. c. on £700	1	8	0
Proof . . .	£648	5	8

EXERCISES.

1. Find the cost of insuring £670; premium 1½ guinea, policy 1*s*., commission ½, per cent.

Ans. £12. 9*s*. 10½*d*.

2. Find the cost of insuring £358. 10*s*., at 35*s*. per cent.; policy 2*s*., commission ½, per cent.

Ans. £8. 9*s*. 4*d*.

3 Find the cost of insuring £1560 at £3 per cent.; policy 4*s*., commission and guarantee 1, per cent.

Ans. £65. 12*s*.

4. Find the cost of insuring £1200 at 3 guineas per cent.; commission ½ per cent., policy 4*s*. per cent., with broker's charge of 5 per cent. on the duty.

Ans. £46. 6*s*. 4½*d*.

5. Find the expense of insuring £5450. 15*s*. on goods per the Fanny, from Jamaica to Greenock, at 3½ guineas per cent.; policy with broker's charge 4*s*. 3*d*., commission and del credere 30*s*., per cent.

Ans. £280. 10*s*. 4½*d*.

6. Required the net cost of insuring £1500 on goods from Brazil to Portugal, at 8 guineas per cent., to return £4 per cent. if the vessel sailed under convoy and

arrived, which was the case; policy 4*s.*, commission ½, per cent. *Ans.* £76. 10*s.*

7. What was the net cost of insuring on 100 pipes of wine, per the Mary Anne, from Lisbon to Leith, valued in the policy at £38 per pipe; premium 6 guineas per cent., to return 2½ per cent. for convoy and arrival, which took place; policy 4*s.* 2½*d.*, and commission ½, per cent.? *Ans.* £171. 7*s.* 11*d.*

8. On what sum must insurance be effected, to cover £958; premium £3½, policy 4*s.*, commission ½, per cent.? *Ans.* £1000.

9. How much will cover £1580; premium 45*s.*, and policy 3*s.* 2*d.*, per cent.? *Ans.* £1619. 2*s.* 5½*d.*

10. What sum must be insured to cover £1500, at 4½ guineas, policy 4*s.*, agent's commission and guarantee 1½, per cent.; broker's commission on the duty 8 per cent.? *Ans.* £1599.

11. If the covered interest in a marine insurance be £10 for every 9 guineas worth of cargo insured; commission and policy 15*s.* per cent., anticipated charges for recovery of the loss 2½ per cent.; what is the premium? *Ans.* £2½ p. c.

Returns for Short Interest.

It sometimes happens that the amount on which insurance has been effected, exceeds the interest actually shipped, and hence a return of premium is claimed on the amount over-insured.

In rating for this return, the policy premium is diminished by the brokerage of 1*s.* for every pound or guinea, leaving what is called underwriter's premium; and 10*s.* per cent. is also withheld as compensation to the underwriters for their trouble.

The excess of the interest insured above that which is afterwards declared, is called *Short Interest.*

Examp. Insurance was effected on 600 chests of

tea, valued in the policy at £30 per chest, from China to Liverpool, at £9 per cent., policy 5*s.* and commission ½ per cent.; but, upon arrival, it appeared that only 450 chests had been shipped. Find how much must be returned for short interest, after deducting the usual commission and brokerage. Find also how much per cent. this insurance cost the owners.

Premium	£9. 0*s.*
Brokerage	0 9
Underwriter's Prem.	8 11
Underwriter's Com.	0 10

Return Premium £8. 1*s.* on £4500 amounts to £362. 5*s.* the return for short interest. *Ans.*

First cost of the insurance	£1755. 0*s.*
Return for over-insurance	362 5
Net cost of insurance	£1392. 15*s.*

£13500 : £100 :: £1392. 15*s.* : £10. 6*s.* 4*d.* p. c. *Ans.*

EXERCISES.

12. Required the amount of return for short shipment of 20 bags of cotton wool, valued at £8 per bag, premium £4½ per cent.; deducting from the return the usual commission and brokerage. *Ans.* £6. 0*s.* 10*d.*

13. Insurance was effected, at a premium of 40*s.* per cent., on 82 pipes of wine, from Oporto to London, valued at £40 a pipe; but only 50 pipes were shipped. Required the amount of return for short interest, and the proportionate amounts of surrender by the several underwriters, twelve of whom underwrote for £200 each, three for £250 each, and one for £130; deducting the customary commission from the underwriter's premium. *Ans.* Whole return £17. 18*s.* 5*d.*
Each of the twelve £1. 1*s.* 10*d.*; &c.

14. Insurance was effected on 80 tons of flax, valued

at £46 per ton, from Riga to Aberdeen; premium 7 guineas per cent., to return £4½ per cent. for convoy and arrival; policy 4*s*., commission ½, per cent.; but only 60 tons could be received on board. The vessel sailed under convoy, and the flax came safe to its destination. Required the net cost of the insurance, deducting the usual brokerage and commission from the return. *Ans.* £95. 14*s*. 5*d*.

15. An agent at Lloyd's effected an insurance on £1500, on goods from Petersburgh to London, at 4 guineas per cent., commission and policy 15*s*. per cent.; but the quantity shipped being only to the amount of £1172. 6*s*., and the vessel being lost, it is required to find the amount of claim upon the underwriters on account of the loss and short interest; the charges for recovering the loss being £2½ per cent., and the usual brokerage and commission being deducted from the return for over-insurance—for the excess, that is, of the sum insured above the real interest covered.

Ans. £1274. 16*s*. 8*d*.

Note.—The adjustment of *Average Losses*, which forms part of the business of Marine Insurance, is a very complicated subject; the consideration of it is therefore omitted from the present work. For good elementary illustrations of its nature, we refer to Tate's *Counting-House Guide*, Davidson's *Arithmetic Modernized*, Murray's *Commercial Arithmetic*, Morrison's *Accomptant's Guide*, or Carey's *Arithmetic*.

EXCHANGE.

The Rule called EXCHANGE affords practice in calculating how much of the money of one country is equivalent to a proposed sum of the money of another.

The settlement of accounts between parties residing in different countries is generally effected by Bills of Exchange, to avoid the inconvenience of transmitting money or bullion. The commercial intercourse, for instance, between Hamburgh and London, must always involve the existence of pecuniary claims upon each of these places by the other; and if we suppose A. of Hamburgh

owes B. of London £500, and C. of London owes D. of Hamburgh £500; A. of Hamburgh can pay D. of Hamburgh £500, and thus buy an order to receive the same amount from C. of London; A. of Hamburgh can then remit the order to B. of London, authorizing him to receive payment of it from C.

The term of a foreign bill of exchange is regulated by the agreement of the parties, or the custom of the countries. Sometimes a bill is drawn at sight, sometimes at a certain number of days or months after sight or after date, and sometimes at *usance* (viz. the time usual between certain places), or at *double* or *half usance*.

The *Par* of Exchange is the intrinsic value of the money of one country as compared with that of another; and is estimated by the weight, fineness, and market-prices of the metals.

The *Course* of Exchange is the current price actually allowed in one country for the money of another. This rate is seldom at par, but sometimes above, sometimes below par; the fluctuation being produced by the varying state of traffic, or demand for bills. If Lisbon exports to London the same value of goods which London exports to Lisbon, the exchange is at par, the mutual debts of the two places are equal, and bills on London can be procured in Lisbon as conveniently as bills on Lisbon can be had in London. But if the exports from London exceed those from Lisbon, the balance of trade or debt is against Lisbon, and in favour of London; and if Lisbon remit the balance in bills, the increased demand for them in Lisbon enhances their value, and thus makes the course of exchange favourable to London. When, however, the course of exchange runs high to a certain degree against any place, it becomes the interest of parties there to make remittances in specie instead of bills; and recourse to this procedure tends to check the deviation from par.

One country exchanges with another at a variable sum of the money of the one for a fixed sum of the money of another. Thus London exchanges with Paris at £1 sterling for a variable number of francs; and London exchanges with Lisbon at a variable number of pence sterling for 1 milrei. The calculations of Exchange are performed by Reduction or Proportion, adapted to given Tables of Equivalents; and the selection of a few of these tables, with examples of the modes of computing by them, will suffice to make the student familiar with the leading features of this department of Arithmetic.

Exchange with France.

In France accounts are generally kept in *francs* and *centimes*, that is, in *francs* and *hundredths*. They were formerly kept in *livres*, *sous*, and *deniers*.

100 *cents.* = 1 *franc.*; 80 *francs* = 81 *livres.*
12 *deniers* = 1 *sou*; 20 *sous* = 1 *livre*; 3 *livres* = 1 *ecu.*

The *franc* is worth about 9½*d.* sterling.

Exchange with Spain.

In Spain accounts are kept in two different kinds of money called *plate* and *vellon*; the former being, as 32 to 17, more valuable than the latter, and employed in bills of exchange.

34 *maravedis* = 1 *real*; 8 *reals* = 1 *peso, piastre,* or *dollar of exchange*; 4 *piastres* = 1 *pistole of exchange*; 375 *maravedis* = 1 *ducat of exchange.*

The *piastre* is worth about 3*s.* 1½*d.* sterling.

Exchange with Portugal.

In Portugal accounts are kept in *reis*, *milreis*, and *old crusados.*

400 *reis* = 1 *crusado*; 2½ *crusados* or 1000 *reis* = 1 *milrei.*

The *milrei* is worth about 4*s.* 6*d.* sterling.

Exchange with Holland and Belgium.

In Holland and Belgium accounts are kept in *florins* or *guilders*, *stivers*, and *pennings*; the *florin* being sometimes divided into 100 cents.; and exchange is also transacted in *schillings* and *grotes* Flemish.

16 *pennings* = 1 *stiver*; 20 *stivers* = 1 *florin*; 2½ *florins* = 1 *rixdollar.*

12 *grotes* or *pence* = 1 *schilling*; 20 *schillings* = 1 *pound Flemish*:—Also 1 *pound Fl.* = 6 *florins.*

The *florin* is worth about 1*s.* 8*d.* sterling.

Exchange with Hamburgh.

At Hamburgh accounts are kept in *marks*, *schillings*, and *pfenings*; also in *pounds*, *schillings*, and *grotes* Fl.

12 *pfenings* = 1 *schilling*; 16 *schillings* = 1 *mark* = 32 *grotes* Fl.; 2 *marks* = 1 *dollar of exchange*; 3 *marks* = 1 *rixdollar*.

In Hamburgh, as used to be the case in Amsterdam, there are two kinds of money, *banco* or bank money, and *currency*. Exchanges are computed in *banco*, which bears a premium, called *agio*, generally from 20 to 25 per cent., against *currency*.

A *mark banco* is worth about 1*s*. 5½*d*. sterling.

Examp. Reduce £470. 12*s*. sterling to francs, exchange 25 frs. 50 cts. per £ sterling.

£470·6 × 25·5 f. = 12000 frs. 30 cts. *Ans.*

Examp. How much sterling in 4795 frs. 46 cts., exchange 24 frs. 45 cts.?

4795·46 f. ÷ 24·45 f. = £196. 2*s*. 8*d*. *Ans.*

Examp. Convert 15374 reals 20 maravedis, into francs, exchange 16 livres 4 sous per pistole of exchange.

15374·59 re. ÷ 32 re. quotes pistoles; these × 16·2 liv. are converted into livres; and the livres × 80 ÷ 81 become francs; therefore,

$$\frac{15374{\cdot}59 \times 16{\cdot}2 \times 80}{32 \times 81} = \frac{15374{\cdot}59}{2} = 7687 \text{ frs. } 29 \text{ cts. } \textit{Ans.}$$

Examp. If the pay of a British seaman be £2. 10*s*. a month, and that of a Dutch seaman 20 guilders; what is the difference in British money, reckoning the exchange at 35*s*. 6*d*. Flemish per £ sterling?

426*d*. Fl. ÷ 40 = 10·65 guilders; 20 gu. ÷ 10·65 gu. = £1. 17*s*. 6¾*d*., which is less than £2. 10*s*. by 12*s*. 5¼*d*. *Ans.*

Examp. Hamburgh remits to London £63. 9*s*. 6*d*.

Flemish, when the rate of exchange is 13 mks. 7 sch. banco per £ sterling. Required the sterling amount.

$13\frac{7}{16}$ mks. × 32*d.* Fl. = 430*d.* Fl.; 15234*d.* ÷ 430*d.* = £35. 8*s.* 6¾*d.* *Ans.*

Examp. How much sterling in 785 mks. 9 sch. Hamburgh currency, at the rate of 34*s.* 6*d.* Fl. per £ sterling, agio 22 per cent.?

Since 100 mks. banco = 122 mks. currency, therefore,
122 : 100 :: 785·5625 mks. cur. : 643·9037 mks. bo.

Now 34*s.* 6*d.* Fl. = 414 grotes, which, at 32 grotes to the mark, is 12·9375 mks.

12·9375 m. : 643·9037 m. :: £1 : £49. 15*s.* 4¾*d.* *Ans.*

Examp. London remitted to Madrid £1890 when the current exchange was 35*d.* per piastre, and drew for reimbursement at 37½*d.* per piastre. Required the gain or loss.

London evidently gains 2½*d.* on 35*d.*; therefore,
35*d.* : 2½*d.* :: £1890 : £135 gain. *Ans.*

EXERCISES.

1. Remitted from Rouen to London 4732 frs. 70 cts.; required the amount sterling, at the exchange of 24 frs. 60 cts. per £. *Ans.* £192. 7*s.* 8¾*d.*

2. Convert £523. 15*s.* British into French money, exchange 25 frs. 85 cts. *Ans.* 13538 fr. 93¾ cts.

3. Represent £346 sterling in Spanish money, exchange 34*d.* sterling per piastre.
Ans. 2442 pi. 2 re. 28 mar.

4. Reduce 1000 reals vellon to sterling, exchange 36½*d.* per dollar of plate. *Ans.* £10. 1*s.* 11¾*d.*

5. In £854. 16*s.* 9*d.* sterling, how much Portuguese, exchange 4*s.* 6*d.* sterling per milrei?
Ans. 3799 mil. 278 reis.

6. When the exchange between Paris and Amster-

dam is 56 grotes for 3 francs, what is the exchange in francs per 100 florins? *Ans.* 214 fr. 28¼ cts.

7. How many Dutch florins in £260. 11*s.* 3*d.* sterling, exchange 12 fl. 7 stiv. per £ sterling?
Ans. 3217 fl. 18 st. 15 pen.

8. How much sterling in 750 florins, exchange 35*s.* 2*d.* Flemish per £ sterling? *Ans.* £71. 1*s.* 9½*d.*

9. How many pounds Flemish in £670 sterling, exchange 11 fl. 10 stiv. per £ sterling?
Ans. £1284. 3*s.* 4*d. Fl.*

10. Reduce £426. 12*s.* sterling to Hamburgh banco, exchange 13 mks. 8 sch. banco per £ sterling.
Ans. 5759 mks. 1 sch. 7 pf.

11. In £257. 13*s.* 6*d.* sterling how much Hamburgh currency, exchange 13 mks. 12 sch. banco per £ sterling, agio 18 per cent.? *Ans.* 4180 mks. 12 sch. 5 pf.

12. Required the sterling worth of the Hamburgh mark current, when the exchange is 33*s.* 6*d.* Flem., agio 21 per cent. *Ans.* 1*s.* 3¾*d.*

13. Remitted from London to Hamburgh £63. 12*s.* 6*d.* to discharge accounts for repairs of a vessel; how many marks current should be received for that sum, exchange 35*s.* 3*d.* Flem., agio 20 per cent.?
Ans. 1009 mks. 4 sch.

14. If the par of exchange between London and Amsterdam be 12 florins per £ sterling; what is the gain or loss per cent. to London, when the current rate is 12 flor. 45 cents.? *Ans.* 3¾ p. c. gain.

15. A merchant at Leith incurs cost of repairs on a vessel at Amsterdam, to the amount of 325 guild. 12 stiv., course of exchange 34*s.* 6*d.* Flem. per £ sterling; what did the merchant save or lose by delaying remittance of payment till the exchange rose to 35*s.* 4*d.*?
Ans. Saved 14*s.* 10*d.* sterl.

16. An agent in London is ordered to effect insurance on £2500, one-third share of a vessel from Bahia to Oporto. The cost of the insurance is

£170. 10*s.* 6*d.*; agent's commission on sum insured ¼ per cent. on effecting, and ¼ per cent. *del credere*. He values on his employer, quoting 56½*d.* per milrei; what sum in Portuguese money will honour the draft?

Ans. 830 mil. 549 reis.

17. I purchased at Oporto 364 pipes of wine at 91 mil. 650 reis per pipe, and paid for freight, customs, &c., as much as the article cost me at first purchase. For how much sterling must I sell it per pipe, to gain 15 per cent. allowing 3 months credit, exchange 63*d.* per milrei? *Ans.* £56. 0*s.* 6*d.*

18. The Portuguese amount of two drafts, on Lisbon and Oporto, respectively, is 770 milreis; the sum sterling of each is the same; and the former is remitted at 60½*d.* and the latter at 51*d.* per milrei. Required the sterling sum of each draft. *Ans.* £81. 4*s.* 1½*d.*

19. Remitted from London to Seville the proceeds of a sale of Spanish wool, which netted 3*s.*, 2*s.*, and 1*s.* 9*d.*, sterling, per lb., according to three qualities varying in quantity as 6, 2, and 1. The net proceeds amounted to 7862 doll. 7 re. 5 mara., exchange 38*d.* sterling per dollar. What quantity was there of each quality.

Ans. First quality 6290$\frac{1}{3}\frac{1}{7}\frac{7}{0}$ lbs., &c.

20. A merchant in London purchases there a quantity of cochineal at 4*s.* 5*d.* per lb., which he ships for Hamburgh; shipping charges 2½ per cent. The article sells at Hamburgh at 3 mks. 15 sch. banco per lb. of Hamburgh; charges there 6 per cent. Now, allowing for loss of interest, &c., 2 per cent. on the proceeds of sale, and reckoning 25 English lbs. = 23 lbs. of Hamburgh; what exchange per £ sterling is thus yielded by Hamburgh? *Ans.* 14 mks. 11·88 sch.

Arbitration of Exchange.

Arbitration of Exchange is the process of determining what value will result, by remitting money from one place to another through one or more intermediate places of exchange.

A circuitous remittance is frequently more advantageous than a direct one. A merchant in London may have credit to a certain amount in Amsterdam; and such may be the course of exchange, that if an agent in Paris draw for the amount on Amsterdam and then remit it to London, the London merchant will receive a larger sum than should have been received directly from Amsterdam.

In estimating the merits of any medium of remittance, the rate of exchange must be valued *at short* (i. e. at sight or three days after sight), when bills from one place on another are for a term of credit; and it must also be considered whether an advantage will remain after allowance for agent's commission, &c.

Examp. If London is to receive from Amsterdam 490 florins, when the course of exchange is 12½ florins per £ sterling; required whether it will be more profitable that Paris should draw on Amsterdam and remit to London, exchange between Paris and Amsterdam 209½ francs for 100 florins, and between Paris and London 25 frs. 42 cts. per £ sterling; allowing ½ per cent. commission to Paris.

To compute the value of the indirect remittance, we may proceed thus:—For every 100 florins in 490, Paris allows 209½ francs, therefore $\frac{490}{100} \times \frac{209\frac{1}{2}}{1}$ is the number of francs for which London has credit in Paris; then, for every 25·42 frs. in that number, London will receive £1 *minus* the commission. Therefore,

$\frac{490 \times 209\frac{1}{2}}{100 \times 25 \cdot 42} = £40 \cdot 3836$; which, deducting the commission, nets £40·1817 as the proceeds of the indirect course; but 490 flor. remitted directly at 12½ flor. per £ produces £40. Accordingly, the indirect remittance is better by 3*s*. 7½*d*. *Ans.*

Such questions as the above belong to the Rule of Compound Proportion, and are similar to the three last questions proposed under that Rule in the Manual. The procedure regulating the statement of the above example may be thus exhibited:—

If £1 be given for 25·42 frs.
how much sterl. for 209½ frs.? 25·42fr. : 209½ fr.
If the result be given for 100 fl., how much sterl. for 490 fl.? 100 fl. : 490 fl. } :: £1
If the second result be given at £100 per £100, how much at £99½ per £100? £100 : £99½

The working out of the terms needs not be exhibited.

Examp. How many florins per £ sterling will be made the arbitrated exchange between Amsterdam and London, by bills on Madrid bought in London, at 37½*d.* per dollar of plate of 272 maravedis, and sold in Amsterdam at 100¾ florins per 40 ducats, each 375 maravedis?

$$\left.\begin{matrix} 40 \times 375 \text{ m.} : 272 \text{ m.} \\ 37\tfrac{1}{2}d. : 240d. \end{matrix}\right\} :: 100\tfrac{3}{4} \text{ fl.} : 11{\cdot}77 \text{ fl. } \textit{Ans.}$$

Examp. T. W. of London has credit in Naples for 1200 ducats, when the direct exchange is 3*s.* 5*d.* sterling per ducat. If, however, he should order the amount to be remitted from Naples to Paris at 4 frs. 25 cts. per ducat, thence to Frankfort at 70 kreutzers for 3 francs, thence to Amsterdam at 4*s.* 6*d.* Flem. for 65 kreutzers, thence to London at 11 fl. 16 st. per £. sterling; whether should he gain or lose, in comparison with the direct exchange, allowing ½ per cent. commission at each place of intermediate remittance?

$$\left.\begin{matrix} 472d. \text{ Fl.} : 54d. \\ 65 \text{ kr.} : 70 \text{ kr.} \\ 3 \text{ fr.} : 4\tfrac{1}{2} \text{ fr.} \\ 1 \text{ du.} : 1200 \text{ du.} \end{matrix}\right\} :: £1 \times \tfrac{199}{200} \times \tfrac{199}{200} \times \tfrac{199}{200}$$

The first of the commission fractions modifies 54*d.*, the second 70 kr., and the third 4½ fr.; but the simplest way of using them is to deduct 1-200th part continuously from £1, which reduces the third term to about £·985.

$$\frac{{\cdot}985 \times 54 \times 70 \times 17 \times 300}{472 \times 65 \times 3} = \frac{197 \times 27 \times 7 \times 17}{236 \times 13} =$$

Indirect remittance. . .	£206	6	2½
Direct, 1200 duc. at 41*d.*	205	0	0
Gain by circuitous exch.	£1.	6*s.*	2½*d.* *Ans.*

EXERCISES.

21. When bills on Madrid are bought in London at 35¾*d.* per peso, and sold in Paris at 15 frs. 17 cts. per

pistole; required the par of exchange between London and Paris. *Ans.* 25 frs. 55 cts.

22. Required the arbitrated rate per peso for bills drawn in London on Madrid, and remitted for sale to Paris; the rate of Paris on Madrid being 15 frs. 60 cts. per pistole, and on London 25 frs. 16 cts. per £ sterling. *Ans.* 37½*d.*

23. What par of exchange is established between Lisbon and Paris, in reis per 3 francs, by bills on London bought in Paris at 25 frs. 73 cts. per £ sterling, and sold in Lisbon at 53*d.* sterling per milrei?

Ans. 528 reis.

24. If 56¾ florins of Amsterdam are given for 120 francs of Paris, 35½ florins of Amsterdam for 40 marks of Hamburgh, 9½ schillings of Hamburgh for 1 bank-note rouble of Petersburgh, and 4 silver roubles of Petersburgh for 15 bank-note roubles; how many francs should be given for 908 silver roubles?

Ans. 3668 fr. 20 cts.

25. A merchant in Amsterdam owes £600 Flemish to a merchant in London. Whether will it be more advantageous for the London merchant to receive the same by direct remittance, exchange 36*s.* 10*d.* Flemish per £ sterling, or by indirect remittance to France, at 56 grotes for 3 francs, thence to Venice at 5 francs per ducat, thence to Hamburgh at 100 grotes per ducat, thence to Lisbon at 50 grotes for 400 reis, and thence to London at 64*d.* sterling per milrei?

Ans. Gain by circuitous remitt. £3. 7*s.*

26. A merchant in Leith owes in Hamburgh 5000 marks curreney. He can order the sum to be drawn on London, exch. 33*s.* 4*d.* Flem. per £ sterling, agio 17½ per cent.; or on Amsterdam, exch. 34 stivers per dollar, commission there ½ per cent., exch. from Amsterdam on London 35*s.* 6*d.* Flem. per £ sterling. Required the cheaper way and the difference.

Ans. On London, cheaper by 18*s.*

27. London has to remit £1000 sterling to Portugal,

when the exchange is 55½*d.* per milrei. Now, if the exchange between Britain and Holland be 12 fl. 12 st. per £ sterling, between Holland and France 50 fl. for 105 frs., and between France and Portugal 490 reis for 3 francs; whether will the direct or the circuitous course be more advantageous for Portugal, allowing ¼ per cent. commission to Holland, and ¼ per cent. to France?

Ans. Direct, by 34 mil. 884 reis.

28. Aymers and Co. of London have credit in Leghorn for 13600 pezze, for which they can draw directly at 50*d.* per pezza; but, wishing to try a circuitous exchange, they order the value to be remitted first to Hamburgh at 33 pezze per 100 marks, thence to Amsterdam at 35 florins per 40 marks, thence to Cadiz at 2½ florins per ducat of 375 maravedis, thence to Lisbon at 630 reis per peso of 272 maravedis, thence to Paris at 525 reis per 3 francs, and thence to London at 10*d.* per franc. Whether do they gain or lose by this speculation, and how much—the expenses of the circuitous remittance being equivalent to 2½ per cent. on the net proceeds? *Ans.* Gain £76. 17*s.* 4*d.*

PURCHASE OF STOCKS.

STOCK is a general name for the capital of a trading company.

The *Stocks* or *Public Funds* represent money borrowed by Government to defray the expenses of the public service. A national creditor or stockholder receives for his loan a half-yearly payment of interest out of the public revenues, but he cannot demand repayment of the capital. He may, however, sell or transfer the whole or a part of his stock to another person; and hence arise the transactions of stock-jobbing, or speculation in the Public Funds, and the employment of transfer agents called stock-brokers, whose charge is 2*s.* 6*d.* or ¼ per cent. for effecting a transfer of stock.

What is termed the Unfunded Debt is constituted by bills

issued by the Exchequer, Navy, Victualling, and Ordnance departments; and it is so called because there is no specific provision, by appropriation of taxes, for its payment. This Unfunded Debt, with the Stocks or Funds, constitutes what is called the National Debt—an amount exceeding 840 millions, and costing about 30 millions for yearly interest and management.

The capital of the Bank of England, and those of the East India and South Sea Companies, being transferable like government funds, are also denominated Stock.

The national debt is vested in several funds, some of which bear 3, some 3½, some 4, and some 5, per cent. A large proportion of it is in two Three per cent. funds, viz., the 3 per cent. consols. or consolidated annuities, so termed because they have been formed by uniting funds that were once distinct, and the 3 per cent. reduced annuities, which derive their name from the reduction of their original rate of 4 per cent.

The holder of stock receives its interest at certain half-yearly terms. The dividends on the 3 per cent. consols. and several other stocks are payable on the 5th of January and 5th of July; those on the 3 per cents. reduced, &c., on 5th of April and 10th of October. The price of stock, therefore, rises as the period for payment of interest approaches, and falls immediately after interest has been paid; so that the 3 per cent. consols. and the 3 per cents. reduced can never sell at the same prices. There is also a fluctuating value in stock, arising from opportunities for investment of capital in other speculations, from the plenty or scarcity of money, &c. If, for instance, I wish to sell £100 of 3 per cent. stock, when capitalists can procure 4 per cent. elsewhere, I must not expect them to offer more than £75 for my nominal £100.

Examp. Sold out £825 India Stock, when the price was 195$\frac{3}{8}$, brokerage $\frac{1}{8}$, per cent.; what did I receive?

That is, if £195$\frac{1}{4}$ be received for £100 stock, what is received for £825?

100 : 825 :: £195$\frac{1}{4}$: £1610. 16*s.* 3*d.* *Ans.*

Examp. What must be paid for £360 four per cent. stock, at 124$\frac{3}{4}$, brokerage $\frac{1}{8}$?

That is, if £124$\frac{7}{8}$ is paid for £100 stock, how much is paid for £360?

100 : 360 :: £124$\frac{7}{8}$: £449. 11*s.* *Ans.*

Examp. How much stock in the 5 per cents., at 83$\frac{1}{2}$, can be bought for £557. 10*s.*, brokerage $\frac{1}{8}$?

83$\frac{1}{2}$: 557$\frac{1}{2}$ :: £100 stock : £666. 13*s.* 4*d.* *Ans.*

Examp. What rate of interest arises from money invested in the 3 per cents. at 48?

£48 purchases £100 stock, or £3 per annum; therefore, 48 : 100 :: £3 : £6¼ p. c. per annum. *Ans.*

Examp. What yearly income arises from £2500 invested in the 3 per cent. consols, at 93¼?

93¼ : 2500 :: £100 stock : £2680·965 stock; 3 per cent. on which is 26·80965 × 3 = £80. 8*s*. 7*d*. *Ans.*

Examp. The 3 per cents. being at 74¼, and the 4½ per cents. at 94⅞; which stock is the more eligible for investment?

74¼ : 94⅞ :: £3 : £3. 16*s*. 8*d*.

The interest from £94⅞ invested in the 3 per cents. is £3. 16*s*. 8*d*., and from the same sum in the 4½ per cents. it is £4. 10*s*.

Hence, the 4½ per cents. are better by 13*s*. 4*d*. per annum on every £94⅞ invested, or by 14*s*. 0½*d*. per cent. per annum. *Ans.*

Examp. I invested £1001 in the 3 per cents. at 89⅜, and having received a whole year's dividend upon it, I sold out, and thereby made my whole increase of capital 72 guineas. Required at what price I sold the stock.

£1001 : £1076. 12*s*. :: 89⅜ : 96⅛ value of £100 stock at the year's end; therefore I must have sold at 96⅛ − 3, or 93⅛. *Ans.*

EXERCISES.

1. If £154 will purchase £100 of Bank Stock, what money will purchase £2050 of the same stock?

Ans. £3157.

2. How much 4 per cent. stock, at 80⅛, can be purchased for £1200? *Ans.* £1497. 13*s*. 2¼*d*.

3. For what amount of stock in the 3 per cent. consols. may I obtain £360, when the price is 92⅜, brokerage ⅛? *Ans.* £390. 4*s*. 10½*d*.

4. How much stock in the 3 per cent. reduced annuities can I purchase for £3380, when the price is 63½, brokerage ¼? *Ans.* £5333. 6*s.* 8*d.*

5. What must I pay for £1350 Navy 5 per cents., when the price is 115¼ per cent., brokerage ⅛? *Ans.* £1560. 18*s.* 9*d.*

6. Sold £3420 Danish 3 per cent. stock at 87; required the proceeds. *Ans.* £2975. 8*s.*

7. What rate of interest arises from money invested in India Stock at 251, the dividends being 10½ per cent. per annum, brokerage ⅛? *Ans.* £4. 3*s.* 7½*d.*

8. At what do the 4 per cents. sell, when they yield 4⅓ per cent. on investments? *Ans.* 90.

9. When the 4 per cents. sell at 86, at what should the 3 per cents. sell to yield the same interest on investments? *Ans.* 64½.

10. I have laid out £1950 in the purchase of 3½ per cent. stock, at 68½, brokerage ½. Required my half-yearly income. *Ans.* £50.

11. What sum must I lay out in the purchase of 3½ per cents. at 88⅝, that my income may be £100 a year? *Ans.* £2525.

12. At what is 3 per cent. stock valued, when purchasers have 4½ per cent. for their investments. *Ans.* 66⅔.

13. In what description of stock, bought at 82½, would £916. 13*s.* 4*d.* purchase an annuity of £50? *Ans.* 4½ per cents.

14. When the 3 per cents. are at 72¼, and the 5 per cents. at 118¾, which stock is preferable for investment? *Ans.* The 5 per cents. by 1*s.* 5*d.* p. c. per ann.

15. If £4000 in the 3 per cents. reduced be sold out at 64½, what sum in addition to the proceeds will procure an annuity of £300 from the Navy 5 per cents. at 97¼? *Ans.* £3307. 10*s.*

16. If £700 stock in the 3 per cents. is sold out at 64½, and the proceeds are invested in the 4 per cents.

at 81½, what improvement in the half-yearly dividend does the transfer produce? *Ans.* 10*s.* 8½*d.*

17. If I invest £5313 in the 3 per cents. at 80½, and, on the stock rising to 83$\frac{3}{4}$, transfer it to the 3½ per cents. at 96½, what increase do I thereby make in my yearly income? *Ans.* 2 guineas.

18. At what are the Dutch 4 per cents. and the Belgian 5 per cents. respectively selling, when both yield the same income, and the sum of the prices of both is 171? *Ans.* D. 76; B. 95.

19. I sold out £1000 of 3 per cent. stock, at 92½, and invested the proceeds, for 18 months, in another speculation, which cleared 6½ per cent. per annum. I then laid out the amount in the 3½ per cents., and found my yearly income £13. 1*s.* better than at first. At what price did I buy into the latter stock? *Ans.* 82$\frac{5}{14}$.

20. A. buys a certain amount of stock at 74½, and, shortly afterwards, sells out to B. at 81, gaining thereby £112. 10*s.* The income is £100 a year. Required the rate of interest. *Ans.* 6 per cent.

21. A stock-jobber purchased £3100 stock in the 3 per cents. when they were at 61$\frac{1}{8}$. Immediately on receipt of a half-year's dividend, a report affecting the expediency of the investment induced him to sell out £1650, the price being 54¼; but, after receipt of his next dividend, he sold the remaining stock at such a price, that his gain upon the whole was £13. 6*s.* 3*d.* He paid the usual brokerage for each transaction. At what price was the stock selling when he closed the speculation? *Ans.* 65½.

Alligation, or Mercantile Compositions.

Alligation is the method of comparing the worth of a mixture with the several values of its ingredients.*

* "Many of our late writers on Arithmetic have considered the Rules of Alligation unworthy of notice. Others have con-

The older writers on Arithmetic divide this Rule into four Cases, under the titles of *Medial*, *Alternate*, *Partial*, and *Total* Alligation.

Alligation *Medial* proposes to mix given quantities at given rates; and it requires to find the medium or average rate, that is, the rate of the compound.

Alligation *Alternate* gives the rates of several ingredients, and the rate of the compound; to find what several quantities will suit the given conditions.

Alligation *Partial* gives the rates of several ingredients, and the rate of the compound, together with the quantity of one ingredient; to find what quantities must be taken of the other ingredients.

Alligation *Total* gives the rates of several ingredients, and both the quantity and the rate of the compound; to find each particular quantity.

The name *Alligation* (from the Latin, *Ligo*, to bind) is descriptive of the method usually employed in the solution of questions belonging to this Rule; the rates being linked together, or connected by lines.

We consider it expedient, however, to discard the practice of linking, and to perform the solution by reasoning from first principles. We shall here give four examples in the four cases above mentioned.

Examp. A corn-dealer mixes 20 bushels of grain, worth 5*s.* a bushel, with 36 bushels at 3*s.* 3*d.* Required the worth of a bushel of this mixture.

20 bushels at 5*s.* 0*d.* ....100*s.*
36 at 3 3 117

The 56 bushels are worth ....217*s.*

Or, 217 ÷ 56 = 3*s.* 10½*d.* per bushel. *Ans.*

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descended to give them a place, while they represent them as worthless in business, and unwieldy in practice by any engine less powerful than Algebra. But to me these Rules certainly appear in a very different point of view. Perhaps at no former period was the practice of forming compositions of goods of different qualities more generally prevalent than it is at present, &c."—Murray's *Commercial Arithmetic*, p. 379.

Examp. A corn-dealer wishes to mix one sort of grain, worth 5*s.* a bushel, with an inferior sort at 3*s.* 3*d.*, so as to make the mixture worth 3*s.* 10½*d.* What quantities shall he take for that purpose?

This question admits an infinite number of different sets of answers; but, as only two quantities are required, the *ratio* of these quantities will be determinate; and therefore, let us inquire, what *proportions* at 5*s.* and 3*s.* 3*d.* will form a mixture at 3*s.* 10½*d.*

The mixing price is an advance of 7½*d.* a bushel on the inferior sort, and a balancing depression of 13½*d.* a bushel on the superior. Consequently, the inferior sort computed at 7½*d.* must equal, in amount, the superior computed at 13½*d.*; or, the quantities of superior and inferior are as 7½ and 13½ respectively, or as 5 : 9. Accordingly, 5 parts of the better to 9 parts of the worse grain must constitute the mixture required. *Ans.*

In verification of this we may try any two quantities in the proportion of 5 : 9, at the respective rates of 5*s.* and 3*s.* 3*d.*; and we shall find the mean rate to be 3*s.* 10½*d.*

Hence, a general Rule for determining the ratio of two quantities, when their separate prices and their average price are given, may be expressed as follows:—

The difference of the average and lower rates, is to that of the average and higher rates, as the quantity at the higher rate to the quantity at the lower rate:—both differences being expressed in one denomination.

Examp. How much corn at 3*s.* 3*d.* a bushel must be mixed with 22½ bushels at 5*s.* a bushel, that the mixture may be worth 3*s.* 10½*d.* a bushel?

Superior : Inferior :: 46½*d.*—39*d.* : 60*d.*—46½*d.*

Therefore, 7½ : 13½ :: 22½ bu. : 40½ bu. *Ans.*

Examp. How many bushels at 5*s.* and at 3*s.* 3*d.* must be mixed together to form a composition of 98 bushels, worth 3*s.* 10½*d.* a bushel?

As previously obtained, we have the quantities of superior and inferior as 7½ : 13½, or as 5 : 9; there-

fore, 98 bushels divided into two parts, as 5 : 9, gives 35 bushels at 5*s.* and 63 bushels at 3*s.* 3*d.* *Ans.*

*** To one or other of these four exemplifications, all questions in Alligation may be conformed.

Examp. In what proportion must sugars, worth 13*d.*, 11½*d.*, and 9*d.* per lb., respectively, be compounded, that the mixture may be worth 10½*d.* per lb.?

First, let the rates 13*d.*, 11½*d.*, and 9*d.*, be reduced to two rates, by arbitrarily choosing certain proportions of quantity for two of them; say, the quantities at 11½*d.* and 9*d.* shall, *if possible*, be *equal*; then, the average price of these will be (11½*d.* + 9*d.*) ÷ 2 = 10½*d.*, between which and 13*d.* the given average, 10½*d.*, *is* possible. Now the problem is reduced to this:—Find proportions at 13*d.* and 10½*d.* to make a mixture at 10½*d.*

Superior : Inferior :: 10½ − 10½ : 13 − 10½; or as ½ : 2½; or as 1 : 10; therefore, the proportion is 1 lb. at 13*d.* to 10 lbs. at 10½*d.*

But, as 10½*d.* is an average price which we assumed for equal quantities at 11½*d.* and 9*d.*, we must now divide 10 lbs. into two equal quantities; and thus we have 1 lb. at 13*d.* to be mixed with 5 lbs. at 11½*d.* and 5 lbs. at 9*d.* *Ans.*

Examp. A wine-merchant wishes to mix four different qualities of brandy, at 32*s.*, 28*s.*, 26*s.*, and 18*s.* per gallon, so as to form a compound worth 24*s.* a gallon. What quantities of each shall he take?

First, let us determine whether the given average, 24*s.*, will allow of the first three rates being assigned to equal quantities.

(32*s.* + 28*s.* + 26*s.*) ÷ 3 = 28*s.* 8*d.*; therefore, as 24*s.* is intermediate to 28*s.* 8*d.* and 18*s.*, it is possible to obtain the solution, by first finding the proportions at 28*s.* 8*d.* and 18*s.* to make a mixture at 24*s.*

Superior : Inferior :: 24 − 18 : 28½ − 24; or as 6 : 4½; or as 9 : 7.

Then, considering that 28*s*. 8*d*. was taken as an average rate for three equal quantities, we must divide 9 into three equal parts; or, 3 gall. at 32*s*., 3 at 28*s*., 3 at 26*s*., and 7 at 18*s*. *Ans.*

Otherwise.

We shall find that we are allowed, by the given rate of the compound, to assume that there shall be equal quantities at 32*s*. and 28*s*., and also equal quantities at 26*s*. and 18*s*.

Average of 32*s*. and 28*s*. = 30*s*.; ditto of 26*s*. and 18*s*. = 22*s*.

We are to find, therefore, the proportions at 30*s*. and 22*s*. to make a mixture worth 24*s*.

Superior : Inferior :: 24 − 22 : 30 − 24; or as 2 : 6; *i. e.* 2 at 30*s*. to 6 at 22*s*.

Then, as 30*s*. is an average rate for two equal quantities, and 22*s*. is the same, we take 1 and 1 instead of 2, and 3 and 3 instead of 6. Thus, we have 1 gall. at 32*s*., 1 at 28*s*., 3 at 26*s*., and 3 at 18*s*. *Ans.*

Otherwise.

We shall find that the composition rate permits us to propose, that for the first three prices the proportion of quantities shall be as 1, 2, and 3.

Average of 1 gall. at 32*s*. + 2 at 28*s*. + 3 at 26*s*. = (32 + 56 + 78) ÷ (1 + 2 + 3) = 27*s*. 8*d*.

We must find, therefore, the proportions at 27*s*. 8*d*. and 18*s*. to make a compound worth 24*s*.

Superior : Inferior :: 24 − 18 : 27$\frac{2}{3}$ − 24;

That is, 18 gall. at 27*s*. 8*d*. to 11 at 18*s*. Whence, by resolution of 18 gall. into parts as 1, 2, 3, we have 3 gall. at 32*s*., 6 at 28*s*., 9 at 26*s*., and 11 at 18*s*. *Ans.*

Many problems, not on the subject of mercantile composition, may be referred to this Rule.

Examp. A gentleman's labourers consist of men at 1*s*. 4*d*. and women at 11*d*. per day; and the amount of

the wages of the whole is the same as if each of them had 1*s.* 2½*d.* Required the number of the men, that of the women being 21.

No. of men : No. of women :: $14\frac{1}{2} - 11 : 16 - 14\frac{1}{2}$;
That is, as $3\frac{1}{2} : 1\frac{1}{2}$, or as 7 : 3; therefore,
3 : 7 :: 21 wom. : 49 men. *Ans.*

Examp. Divide £816 into two sums, so that the legal interest of the one for 6 yrs. 3 mo., added to that of the other for 8 yrs. 6 mo., may equal that of the whole sum for 7 yrs. 2½ mo.

Princ. for $8\frac{1}{2}$ yrs. : Princ. for $6\frac{1}{4}$ yrs. :: $7\frac{1}{24} - 6\frac{1}{4} : 8\frac{1}{2} - 7\frac{1}{24}$;

That is, as 63 : 81; or as 7 : 9; therefore, £816, divided into two parts as 7 : 9, gives £357 for $8\frac{1}{2}$ yrs. and £459 for $6\frac{1}{4}$ yrs. *Ans.*

Many problems in Specific Gravity may be resolved according to the Rule of Alligation.

Examp. Hiero, King of Syracuse, having employed a goldsmith to make him a crown containing 63 ounces of gold, suspected that silver had been mixed with the gold—which was the case; and Archimedes, being appointed to examine the crown, found that its immersion in a vessel, containing water, raised the fluid $6\frac{3}{4}$ cubic inches, and that a cubic inch of gold weighed 11 ounces, and a cubic inch of silver 6 ounces. It is required to determine how much silver had been substituted.

The crown contained $6\frac{3}{4}$ cubic inches, and weighed $9\frac{1}{4}$ ounces per cubic inch.

Required, therefore, how many cubic inches of gold at 11 ounces per cubic inch, and of silver at 6 ounces, must be combined to produce a composition of $6\frac{3}{4}$ cubic inches at $9\frac{1}{4}$ ounces per cubic inch.

In. of gold : in. of silver :: $9\frac{1}{4} - 6 : 11 - 9\frac{1}{4}$.

Therefore, if $6\frac{3}{4}$ in. be divided into two parts as 2 : 1, we shall have $2\frac{1}{4}$ cub. in. of silver; which, at 6 oz. per cub. in., gives $13\frac{1}{2}$ oz. of silver. *Ans.*

EXERCISES.

1. A grocer mixes 21 lbs. of raisins at 6½*d.* per lb. with 35 lbs. at 8½*d.* What is the worth of 1 lb. of the mixture? *Ans.* 7¾*d.*

2. If 16 gallons of ale worth 4*d.* a quart be mixed with 20 gallons at 6*d.* a quart and 28 gallons at 8*d.* a quart; what is the worth of a gallon of the mixture?
Ans. 2*s.* 1½*d.*

3. A silversmith melts 3 kilogrammes of silver, in which the 10th part is copper, with 5 kilogrammes of silver, in which the 8th part is copper. What is the proportion of copper in the metal thus produced?
Ans. 37 parts in 320.

4. A dealer in teas wishes to mix two qualities, at 4*s.* 8*d.* and 6*s.* 6*d.*, so that he may sell the mixture at 5*s.* 4*d.* What proportions must he take?
Ans. 4 lbs. at 6*s.* 6*d.* to 7 at 4*s.* 8*d.*

5. In what proportions must spirits at 9*s.* 9*d.* and 10*s.* 10*d.* a gallon be mingled, that the compound may be worth 10*s.*?
Ans. 3 gall. at 10*s.* 10*d.* to 10 at 9*s.* 9*d.*

6. In what proportion must water be mingled with spirits at 9*s.* 6*d.* a gallon, to reduce the price to 9*s.* a gallon?* *Ans.* 1 gall. of water to 18 of spirits.

7. Required the proportions for mixing pears worth 4*s.* and 5*s.* 6*d.* the bushel, to make the average price 4*s.* 10*d.* *Ans.* 5 bu. at 5*s.* 6*d.* to 4 at 4*s.*

8. If in one mass of silver the 8th part is copper, and in another the 15th part; in what proportion must the two kinds be combined, that the 10th part of the compound may be copper?
Ans. 4 parts of the 1st with 3 of the 2nd.

9. What quantity of nutmegs worth 6*s.* 3*d.* a lb.

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* When one of the simples costs little or nothing compared with the others, such as water in diluted spirits, alloy in gold, &c., it is always reckoned at 0.

must be mixed with 5 lbs. of a finer sort at 7*s.* 6*d.*, to make the average price 7*s.* a lb.? *Ans.* 3½ lbs.

10. How many ounces of gold, of the fineness of 18 carats, must be taken with 4 lbs. 6 oz. of 22 carats fine, to make a composition of 21 carats fine?

Ans. 1 lb. 6 oz.

11. A grocer by mixing sugars at 6½*d.* and 8*d.* per lb. has produced a composition of 1 cwt. which he can sell at 6¾*d.* What simples has he taken?

Ans. 18¾ lbs. at 8*d.*, &c.

12. A grocer wishes to mix sugars at 5*d.*, 6½*d.*, and 8*d.* per lb., so as to make the composition worth 7*d.* per lb. What quantities shall he take?

Ans. 2 lb. at 5*d.*, 2 at 6½*d.*, 5 at 8*d.*;
or, 1 lb. at 5*d.*, 4 at 6½*d.*, 4 at 8*d.*;
or, 4 lb. at 5*d.*, 2 at 6½*d.*, 9 at 8*d.*;
&c., &c., &c.

13. Required, to mix ales at 25*d.*, 23*d.*, 14*d.*, 13*d.*, and 9*d.* per gallon, so that the mixture may be sold at 16*d.*

Ans. 7 gall. of each of the first four, with 11½ gall. of the cheapest.

14. How must we mix the qualities given in the preceding question, to produce the given average by means of equal quantities of the two first with equal quantities of the three last? Also, by means of equals of the three first with equals of the two last?

Ans. 3 gall. of 1st and 2nd with 4 of the others,
or 7 gall. of 5th and 6th with 5 of the others.

15. One mass of silver is 11 oz. 2 dwt. fine, and another is 10 oz. 10 dwt. In what equal proportions must these be mixed with alloy, to produce a mass that shall be 10 oz. fine?

Ans. 25 parts of each quality of silver, with 4 parts of alloy.

16. A goldsmith wishes to combine 15 oz. of gold, worth 77*s.* 9*d.* an ounce, with three other descriptions

at 77*s.* 6*d.*, at 76*s.* 8*d.*, and at 76*s.*, so as to form a mass worth 77*s.* 4*d.* Required the quantities.

Ans. 15 oz. of each of the two first kinds;
4 oz. 7½ dwt. of each of the two last.

17. How may a grocer mix teas at 3*s.* 8*d.*, 4*s.* 4*d.*, and 4*s.* 8*d.* per lb., that he may have a composition of 84 lbs. at 4*s.* 3½*d.*?

Ans. 23½ lbs. at 3*s.* 8*d.*, ditto at 4*s.* 4*d.*, &c.

18. A merchant has 4 qualities of spirits in quantities proportioned as 1, 2, 3, 4; and at 18*s.*, 16*s.*, 15*s.*, and 12*s.* per gallon, respectively. With these he mingles such a quantity of water as makes the average value 11*s.* 9*d.* a gallon. How much per cent. of the mixture is water? *Ans.* 17·83 p. c.

19. A vessel containing 1 cubic foot is filled with a mixture of wine and spirits. The specific gravity of the wine is ·95, of the spirits ·858, and of the mixture ·881. How much of each does the vessel contain?

Ans. 432 cub. in. wine, 1296 of spirits.

20. A, B, and C, join stocks in trade; A's capital is continued for 7 months, B's for 18 months, and C's for 12 months. Now, if C has invested as much as A and B together, and also gets half the total gain; what must be the proportion of the several stocks? *Ans.* 6, 5, and 11.

21. A merchant has four different qualities of tea; the second is one-third dearer than the first, the third is one-eighth dearer than the second, and the fourth is one-sixth dearer than the third. In what proportions shall he mix these qualities, to make a composition of 76 lbs., at a rate one-seventh dearer than the first quality? *Ans.* 55⅓ lbs. of the 1st,
6⅔ lbs. of each of the others.

22. A mass of gold and silver, whose specific gravity is 16, contains 36 cubic inches more of the former than of the latter metal; how many inches of each metal is it composed of, the specific gravity of the gold being 19½, and of the silver 10½? *Ans.* 88 c. in. gold, 52 silver.

23. What principal, improved at 3½ per cent. per ann. for 6 years, will yield an interest equal to the sum of the interests of £27 of the principal for 4 years at 5 per cent., £24 of it for 7 years at 3½ per cent., and the remainder of it for 6 years at 3 per cent.? *Ans.* £70.

24. A hop-dealer buys hops at 45*s.*, 45*s.* 10*d.*, and 44*s.* 8*d.* per cwt., with 6 months credit. In what proportions may he mix these qualities, to sell forthwith at 48*s.* 7*d.* cash, gaining thereby 10 per cent.?

Ans. 9 cwt. at 45*s.*; ditto at 44*s.* 8*d.*; 14 at 45*s.* 10*d.*

INVOLUTION.

Products considered as resulting from the continual multiplication of a number by itself, are called *powers* of that number. Thus 8 is a power of 2, because $2 \times 2 \times 2 = 8$; also, $30\frac{1}{4}$ is a power of $5\frac{1}{2}$, because $5\frac{1}{2} \times 5\frac{1}{2} = 30\frac{1}{4}$. The number continually multiplied is called the *root* of the power, and is said to be *involved* or *raised* to the power.

Powers are denoted by small figures, called *exponents* or *indices*, which indicate the number of times the root is used as a factor; thus $2^3=8$ means $2\times2\times2=8$, or, that the 3rd power of 2 is 8.

The 2nd power of a number is usually called the *square* of that number, and the 3rd power of it is called its *cube*. But, properly, the terms square and cube are names of *geometrical*, not *arithmetical*, quantities.

From an inspection of the equations, $5\times5\times5\times5\times5\times5 = 5^2 \times 5^2 \times 5^2 = 5^3 \times 5^3 = 5^3 \times 5^2 \times 5 = 5^5 \times 5 = 5^6$, it will readily appear that the *addition* of the indices corresponds to the *multiplication* of the quantities. Thus the 3rd power of any quantity multiplied by the 3rd power of that quantity produces the 6th power of the same quantity; and the continuous multiplication of the 3rd, 2nd, and 1st powers, produces also the 6th power.

Accordingly, the process of *Involution* will often admit the expedient of abridging the number of factors.

Examp. Involve $4\frac{1}{2}$ to the 10th power.

$$\begin{array}{rl} 4{\cdot}5 & = \text{1st power.} \\ 4{\cdot}5 & \\ \hline 20{\cdot}25 & = \text{2nd power.} \\ 20{\cdot}25 & \\ \hline 410{\cdot}0625 & = \text{4th power.} \\ 410{\cdot}0625 & \\ \hline 168151{\cdot}25 & = \text{8th power.} \\ 20{\cdot}25 & \\ \hline \textit{Ans.}\ 3405062{\cdot}89 & = \text{10th power.} \end{array}$$

The same result would have been obtained by multiplying the 4th power by the 2nd, and then the result (or 6th power) by the 4th. Otherwise, since $4\frac{1}{2} = \frac{9}{2}$, therefore, $9^{10} \div 2^{10} = \frac{3486784401}{4\times4\times4\times4\times4} = \textit{Ans.}$

Recourse may be had to *Logarithms*, to avoid the tedious labour of involution for high exponents.

The manner of computing by Logarithmic Tables needs not be detailed here, as most books which contain them will be found sufficiently explanatory of their use.

EXERCISES.

1. $4^4 = 256$
2. $3^7 = 2187$
3. $6^9 = 10077696$
4. $23^5 = 6436343$
5. $(\frac{5}{8})^6 = \frac{15625}{262144}$
6. $(1\frac{1}{4})^9 = 58{\cdot}432142$
7. $3{\cdot}2^8 = 10995{\cdot}116278$
8. $1{\cdot}05^{13} = 1{\cdot}8856491423$
9. $1{\cdot}0425^{16} = 1{\cdot}9463324272$
10. $\cdot448^{15} = \cdot0000058772$
11. $1{\cdot}03^{48} = 3{\cdot}4606958935$
12. $5 \div {\cdot}02^7 = 3906250$ *mill.*
13. $1{\cdot}4^{10} \times (\frac{5}{14})^{15} = {\cdot}00000567427.$
14. $\frac{1{\cdot}02^{18}}{1{\cdot}02^9} + \frac{1{\cdot}03}{1{\cdot}03^{20}} = 1{\cdot}6527181868.$

EVOLUTION.

The process of finding any root of a number is called *Evolution*, or *extracting the root*.

The root of a number is denoted by giving that number a fractional exponent. Thus $8^{\frac{1}{3}} = 2$ means that the 3rd root of 8 is 2. So also, $8^{\frac{2}{3}}$ signifies the 3rd root of 8^2, or the 2nd power of $8^{\frac{1}{3}}$, viz. 4. Further, $6^{\frac{3}{4}}$ indicates $6^2 \times 6^{\frac{1}{4}}$, or the 4th root of 6^9, or the 9th power of $6^{\frac{1}{4}}$.

Instead of a fractional index, we often find the sign $\surd$ (a form of the letter *r*, denoting *root*) prefixed to the number whose root is to be expressed; thus $\sqrt[4]{81} = 3$. For the square root, which is of most frequent occurrence, we do not express the index with the sign; thus $\surd 81 = 9$.

Extraction of the Second or Square Root.

The method of evolving a root from a power may be deduced from a particular method of involving the root to the power. Let it be required to involve 273, or 200 + 70 + 3, to the second power.

The algebraic involution of $x + y + z$ suggests the following process:—

200	× 200 =	40000
(200 × 2 + 70)	× 70 =	32900
(200 × 2 + 70 × 2 + 3)	× 3 =	1629
	273^2 =	74529

Here the first product is the square of the multiplier 200; the sum of the first and second products is the square of (200+70); and the sum of the three products is the square of (200+70+3). Hence, by a reference to the above composition of a square number, we can readily obtain a method of extracting the root of a square number.

Examp. Extract the square root of 74529.

0	74529	(200	
200	40000	70	
		3	
200*	34529		
200	32900	273	*Ans.*
400	1629		
70	1629		
470*			
70			
540			
3			
543*			

In this process, the numbers marked with an asterisk are the several multiplicands occurring in the previous example of involution, and they are generated according to the form ingeniously devised by Mr. Horner for evolution generally. The solution is effected by means of two columns, the one headed by a cipher, the other by the given number. We determine easily, in the first place, the figure of highest rank in the root, viz. 2 *hundreds;* we therefore write 200 as a part of the answer, on the right hand. To the 0 we add 200, and multiply the sum by the quotient, placing the product under the given number and subtracting. The second addition of 200 in the left column, obtains 400 as a partial or trial divisor for 34529. Now, 400 quotes 80 times; but, as 400+80 will not quote 80 times, we must write only 70 as the second part of the answer; at the same time we complete the partial divisor by adding 70; and then we multiply 470 by 70, and subtract as before. The second addition of 70 in the left column, obtains 540 as a partial or trial divisor for 1629. Then, 540 quotes 3 times, and 540+3, the complete divisor, also quotes 3 times; so that 3 is the third portion of the answer; and when 543 is multiplied by 3, we find that the product exactly completes the square, and that the second root of the proposed number is exactly 273.

The above process may be simplified by suppressing the ciphers, and at once doubling the partial root for a trial divisor. The following, therefore, is the usual and most convenient method of extracting the square root:—

Beginning at the place of units, mark off, from right to left, as many periods of two figures each as the given number allows. Mark off decimals (if any) from left to right. Under the left hand period write the greatest square number it contains, and place the root of that number in the quotient. Subtract the square number from the left hand period; to the remainder annex the next period for a dividend; and, for its trial divisor, double the quotient figure. Consider what is the greatest number of times which, multiplied by the sum of that number and ten times the trial divisor, will yield a number contained in the dividend; this will supply a second quotient figure, which, being annexed to the trial divisor, forms the complete divisor. Multiply the complete divisor by the quotient figure; subtract the product from the dividend; to the remainder annex the next period, for a new dividend; and, for a new trial divisor, either double the quotient, or increase the last complete divisor by the last quotient figure. Continue the operation till all the figures in the given number have been used; and, if there be any remainder, annex periods of two ciphers to find a decimal continuation of the root to the extent required.

	7,45,29 (273 *Ans.*
	4
	—
47	345
7	329
	—
543	1629
	1629
	—

Most numbers are not exact squares; but we can always obtain the root with a degree of accuracy sufficient for practical purposes. In the two following examples the exact roots are interminate quantities.

Examp. Evolve $\sqrt{4809 \cdot 523}$, and $\sqrt{\cdot 001}$.

	48,09·52,30(69·3507+		·00,10(·031623−
	36 *Ans.*		9 *Ans.*
	—		—
129	1209	61	100
	1161		61
	—		—
1383	4852	626	3900
	4149		3756
	—		—
13865	70330	6322	14400
	69325		12644
	—		—
1387007	10050000	6324	1756

When it is proposed to extract the square root of a

vulgar fraction, evolve numerator and denominator separately, if they are exact squares; thus, $\sqrt{\frac{1}{4}} = \frac{\sqrt{1}}{\sqrt{4}} = \frac{1}{2}$; also, $\sqrt{1\frac{7}{9}} = \frac{\sqrt{16}}{\sqrt{9}} = \frac{4}{3} = 1\frac{1}{3}$. Otherwise, reduce to the decimal form, and then extract; thus, $\sqrt{\frac{3}{8}} = \sqrt{\cdot 375} = \cdot 61237$; or first multiply both terms of the fraction by the denominator, making $\sqrt{\frac{3}{8}} = \frac{\sqrt{24}}{\sqrt{64}} = 4\cdot 89898 \div 8 = \cdot 61237$. Other methods may be resorted to; thus, $\sqrt{\cdot 3} = \sqrt{\frac{6}{10}} = \sqrt{6} \div 4$; or, $\sqrt{\frac{3}{8}} = \sqrt{\frac{6}{16}} = \frac{\sqrt{6} \times \sqrt{24}}{24}$.

It may be observed, that, as the successive divisors in evolution tend more and more to uniformity in their left-hand figures, we might, after proceeding a little way with the usual operation, obtain additional figures by means of one divisor, working according to the contracted form of decimal division. Thus, in the second example, the last dividend, when divided by 1387007, will yield the additional figures of the more accurate root, 69·35072458.

The Exercises in § 22 of the Manual will serve as a Praxis in simple Evolution; and it is recommended that several of them be resolved by Horner's method, to prepare the student for the application of that method to the cube root, and to Evolution generally. In proceeding to the subject of Evolution, it is requisite that the pupil be able to name readily the 2nd and 3rd powers of the nine digits.

Extraction of the Third or Cube Root.

Let it be required to involve to the third power the number 546, or 500 + 40 + 6.

The algebraic involution of $x + y + z$ suggests the following method of composition.

500^3	$\times\ 500 =$	125000000
$[500^2 \times 3 + (500 \times 3 + 40) \times 40]$	$\times\ 40 =$	32464000
$500^2 \times 3 + (500 \times 3 + 40) \times 40$ $+ (500 \times 3 + 40 \times 2) \times 40$ $+ (500 \times 3 + 40 \times 3 + 6) \times 6$	$\times\ 6 =$	5307336
	$546^3 =$	162771336

Here, the first product is the cube of 500; the sum

of the first and second products is the cube of (500 + 40); and the sum of the three products is the cube of (500 + 40 + 6).

By reference to this mode of involution we obtain a form for the extraction of the cube root.

Examp. Extract the cube root of 162771336.

0	0	162771336	(500	
500	250000	125000000	40	
			6	
500	250000*	37771336		
500	500000	32464000	546	*Ans.*
1000	750000	5307336		
500	61600	5307336		
1500	811600*			
40	63200			
1540	874800			
40	9756			
1580	884556*			
40				
1620				
6				
1626				

This solution is performed by means of three columns, each of the two first being headed by a cipher, and the third by the given number. We determine, in the first place, the figure of highest rank in the root, viz. 5 *hundreds;* and we write 500, as a part of the answer, on the right hand. To the cipher at the head of the first column, we add the first quotient number, and multiply the sum by that number, setting the product under the cipher at the head of the second column, and adding; we then multiply the sum by the quotient number, setting the product under the head number of the third column, and subtracting for another dividend. Again we add 500 to the first column, multiply the sum by 500, place the product in the second column and add,

for a trial divisor; the next quotient number is readily judged to be less than 50, and we write 40. When we have added the first quotient, for the third time, in the first column, we then increase the sum by the second quotient, multiply the result by 40, place the product in the second column and add, for the complete divisor, and multiply that divisor by 40 for a subtrahend in the third column, which leaves 5307336 as a new dividend. We then add the second quotient to the first column, multiply the sum by that quotient, place the product in the second column and add, for a trial divisor; the next quotient figure is readily discerned to be 6; and, accordingly, after adding 40, for the third time, in the first column, we increase the sum by 6, multiply the result by 6, place the product in the second column and add, for a complete divisor, and multiply that divisor by 6, for a subtrahend in the third column. The figures of the given number being now exhausted, and no remainder arising, we have determined the root to be exactly 546.

The above process admits of being simplified by the omission of ciphers; and the general Rule for the extraction of the Cube Root, by Horner's method, may be stated as follows:—

Place two ciphers and the given number, as the heads of three columns, and divide the given number into periods of three figures —from right to left for integers—from left to right for decimals. Determine the root of the greatest cube contained in the left hand period; write that root as the first quotient figure; place it also under the cipher of the first column and add; multiply the sum by the quotient figure, setting the product under the cipher of the second column and adding; then multiply this latter sum by the quotient figure,	0 5 — 5 5 — 10 5 — 150 4 — 154 4 — 158 4 — 1620 6 — 1626	0 25 — 25 50 — 7500 616 — 8116 632 — 874800 9756 — 884556	162,771,336 (546 125 — 37771 32464 — 5307336 5307336 —

which will give the highest cube to be placed under the left hand period for subtraction. To the remainder annex the next period for a dividend; add the quotient figure to the first column; multiply the sum by the quotient figure, and add the product to the second column; add the quotient figure, a third time, to the first

column, and annex one cipher on the right hand of the sum, and two ciphers on the right hand of the last number in the second column; use this number as a trial divisor, placing the number of times in the quotient, and adding the same figure to the first column. Multiply the sum in the first column by the new quotient figure; add the product to the second column, and multiply its sum by the quotient figure, placing the product under the dividend in the third column and subtracting. Annex the next period, and proceed as formerly, till all the figures of the given number are used, or till, in the case of an interminate root, as many decimals have been obtained as is considered sufficient.

For the evolution of interminate roots we may abbreviate the calculation, after having found two or three figures in the above manner. Thus, in the following example, after having found the figures 1·01, the repetition of annexed ciphers being occasioned by the dividend, 45, not containing the trial divisor, we may take from each of the numbers 14699000, 3060300, and 3020, the three right-hand figures, or, which is the same thing, we may, without annexing the ciphers, take *no* figures from 14699, *one* from 30603, and two from 302. Then, it becomes unnecessary to add quotient figures in the first column; only 4 times 3 need to be placed in the second column, and 4 times 3072 in the third. The sequel of the operation proceeds as in contraction of decimal division.

Examp. Extract the cube root of 1·045.

0	0	1·045(1·01478 *Ans.*
1	1	1
—	—	—
1	1	045000
1	2	30301
—	—	—
2	30000	14699,
1	301	12288
—	—	—
300	30301	2411
1	302	2159
—	—	—
301	3060,3	252
1	12	247
—	—	—
3,02	3072	
	12	
	—	
	30,8,4	

Although the above method of performing the extraction of the Cube Root is distinguished by simplicity and elegance, and is eligible on account of its conformity to Horner's admirable Rule for the general extraction of roots, the following more compressed form will be preferred by some calculators. It is the improved method devised by Mr. Holdred,* an ingenious pioneer of Horner; although the author of this work once thought that it was for the first time exemplified in his own little publication,—*A Supplementary Course of Arithmetical Exercises* (Edin. 1830).

Examp. Required the cube root of 162771336.

$50^2 \times 3 =$	7500	162,771,336(546
$154 \times 4 =$	616	125
	8116	37771
$4^3 =$	16	32464
	874800	5307336
$1626 \times 6 =$	9756	5307336
	884556	

Here, having divided the given number into periods, and determined the root of the greatest cube contained in the left-hand period, writing that root as the first quotient figure;—we take thrice the square of 10 times that root, as a trial divisor by which to ascertain the second quotient figure; then, to form the complete divisor, we annex the second quotient figure to thrice the first, and multiply by the second, adding the product to the trial divisor. To find the next trial divisor, we add together the square of the second quotient figure, the last complete divisor, and the number standing above that divisor, and to the sum of these three annex two ciphers. Then, having by means of this new trial divisor ascertained the third quotient figure, we annex that figure to thrice the preceding part of the quotient, and then multiply by that figure, adding the product to the trial divisor to form the complete divisor. The other parts of the operation are the same as formerly exemplified.

Referring to the Manual for elementary Exercises in the Cube Root, we shall here furnish an example which, with those already given, may serve to indicate the general applicability of Horner's method.

* See a *New Method of Solving Equations*, by Theophilus Holdred, p. 45 (London, 1820). See also Tate's *Principles of Arithmetic*, 2nd ed., p. 98.

Examp. Required the 4th root of 36·8.

0	0	0	$\sqrt[4]{}$36·8000(2·462986
2	4	8	16
—	—	—	—
2	4	8	208000
2	8	24	171776
—	—	—	—
4	12	32000	3622400,00
2	12	10944	3444264
—	—	—	—
6	2400	42944	178136
2	336	12352	119240
—	—	—	—
80	2736	552960,00	58896
4	352	21084	53757
—	—	—	—
84	3088	574044	5139
4	368	21432	4781
—	—	—	—
88	3456,00	595476	358
4	58	73	358
—	—	—	—
92	3514	59620	
4	58	73	
—	—	—	
009,60	3572	59693	
	58	3	
	—	—	
	3630	5973	
		3	
		—	
		5976	

In the above example we have four columns, corresponding to the order of the root required; the first three being headed by ciphers, and the fourth commencing with the given number. The first period is 36; therefore, the greatest whole number whose fourth power is contained in 36 is 2, the first figure of the root; and 16, the fourth power of 2, being subtracted from 36, we annex to the remainder the next four figures of the given number for a dividend.

Now, in the third column, there must always be two additions made, to obtain a trial divisor for a new quotient figure; and these additions are derived from two additions in the second column, which, in like manner, are derived from two additions in the first. The additions in the first are made by repetition of the first quotient figure; those in the second are made by the sums in the first, successively multiplied by the first quotient figure; and those in the third are made by the sums in the second, successively multiplied by the first quotient figure.

The sum now obtained in the third column must have three ciphers annexed; while the first and second columns, being extended until *four* additions have been made in the first and *three* in the second, have one cipher annexed in the first, and two in the second.

We have now the numbers 80, 2400, 32000, and 208000, to work with for the second figure of the root; and the process is continued in a manner quite analogous to that employed for the cube root. After obtaining the second set of ciphered numbers, 960, 345600, 55296000, and the dividend 362240000, we have commenced a contracting process, by cutting off two figures, and making allowance for carriage. (*See* Example of Contraction for the Cube Root.)

*** When roots are of a high order, no method of evolution will be found so convenient as that by Logarithms. It should be observed, however, that when the number denoting the order of a root is a composite number, the root may be found by two or more evolutions. Thus, the 4th root of a number may be found by taking the square root of the square root of that number; the 6th root is = the 2nd root of the 3rd root; &c.

EXERCISES.

1. The 4th root of 16243247601. *Ans.* 357.
2. The 4th root of 1·05. *Ans.* 1·012272.
3. The 4th root of 11. *Ans.* 1·8211603.
4. The 5th root of ·820481533. *Ans.* ·9612.
5. The 5th root of 1·03. *Ans.* 1·0059293.
6. The 5th root of 25456081. *Ans.* 30·28017.
7. The 6th root of 1·32879134. *Ans.* 1·0485186.
8. The 6th root of $\dot{\cdot}\dot{5}206\dot{}$. *Ans.* ·8969287.
9. The 7th root of 43720. *Ans.* 4·602081.
10. The 9th root of 1·045. *Ans.* 1·0049027.
11. The 18th root of 6125. *Ans.* 1·623285.

APPLICATIONS OF THE SQUARE ROOT.

*To find a Mean Proportional.**

If $a : m :: m : c$, then m is called a mean proportional between a and c.

In like manner, since $9 : 12 :: 12 : 16$, we call 12 a mean proportional between 16 and 9; and because $12 \times 12 = 16 \times 9$, therefore, the mean proportional between 9 and 16 is the square root of the product of 9 and 16, or the product of the square roots of those numbers.

Examp. A quantity of goods weighed in the one scale 36 lbs. avoirdupois, and in the other 36 lbs. 2 oz. Required their true weight.

In this problem it is supposed that the beam of the balance is a straight lever of uniform density, and that the fulcrum is not at the middle of the beam: and it is manifest that the weights in the scales a and b are, first, 36 lbs. and the quantity of goods, respectively; and secondly, the quantity of goods and 36·125 lbs., respectively. Accordingly, 36 lbs. must be to the true weight as the true weight is to 36·125 lbs.; or, the true weight is a mean proportional between 36 and 36·125, viz. $6 \times \sqrt{36 \cdot 125} = 36 \cdot 062446$ lbs. *Ans.*

Examp. Suppose a shopkeeper to gain 26 per cent. by buying goods on the shorter and selling them on the longer arm of a fraudulent beam, 17 inches long; of what lengths are the arms of the beam?

100 lbs. on the long arm : a greater quantity on the short arm :: that greater quantity on the long arm : 126 lbs. on the short arm.

Therefore, the mean proportional between £100 and £126, viz. £112·24972, is the amount of £100 by his gain in *buying*. Divide 17 in the proportion of 100 and 112·24972, and the lengths of the arms are found to be 8·00943 and 8·99057 inches. *Ans.*

* This is only a special case of the Rule of *Equirational Series*.

Examp. George and Henry work each 10 hours a day, at different rates of labour. George can do Henry's daily work in $\frac{7}{8}$ of the time that Henry would take to do George's daily work. What is the direct proportion of their daily labour?

As H's daily work *is to* G's daily work, *so is* G's time of doing H's daily work *to* 1 day; and *so is* 1 day *to* H's time of doing G's daily work.

Therefore, 1 day is a mean proportional between H's time of doing G's daily work, and G's time of doing H's daily work. That is, 1 day is a mean proportional between a certain time and $\frac{7}{8}$ of that time.

Consequently, $\sqrt{\frac{7}{8}}$, or ·9354 of that time = 1 day; and hence, H's time of doing G's daily work is 10000...9354ths of a day; or G's rate is to H's rate as 5000 : 4677. *Ans.*

Examp. A and B set out to meet each other, A leaving the town C at the same time that B left D. They travelled the direct road C D; and, on meeting, it appeared that A had travelled 18 miles more than B, and that A could have gone B's journey in 15¾ days, but B would have been 28 days in performing A's journey. What was the distance beween C and D?

A's time to the longer journey : A's time to the shorter (15¾ days) :: B's time to the longer journey (28 days) : B's time to the shorter.

Here the first and last terms refer to the actual journeys of A and B, respectively, for which the times are equal; therefore, $\sqrt{28 \times 15\frac{3}{4}} = 21$ days, duration of each journey.

28 − 21 : 28 + 21 :: 18 mi. :: 126 mi. *Ans.*

EXERCISES.

1. A rectangular space is 12½ feet long and 11 feet broad. Required the side of a square space equally large. *Ans.* 11·72604 ft.

2. The axes of an ellipse are 526 and 354 inches. What is the diameter of a circle of equal superficies?

Ans. 431·513615 in.

3. A sells a quantity of tea to B for £231. 4*s*., and B sells it to C, who disposes of it for £255. 12*s*. 3*d*. What does C pay for it, if each person gains at the same rate per cent.? *Ans.* £243. 2*s*.

4. A sets out from London to York, and B from York to London, at the same time. A arrives in York 9¼ hours, and B in London 14¼ hours, after they meet. In what time does each perform the journey?

Ans. A 21·2367 hrs.; B 26·2367 hrs.

5. Two travellers, M and N, set out at the same time from two places, G and D;—M from G, with a design to pass through D, and N from D, to travel in the same direction. After M had overtaken N, they found that they had, both together, travelled 30 miles; that M had passed through D 4 days before; and that N, at his rate of travelling, was a journey of 9 days distant from G. Required the distance between the places G and D.

Ans. 6 miles.

6. A merchant has a fraudulent balance:—He buys in on the shorter arm, and sells out on the longer, and thereby gains 5·19395 per cent. But if the longer arm measured half an inch more, the shorter remaining unaltered, his gain would be 11·8713 per cent. Required the length of each arm of the beam.

Ans. 16 in. and 15·6 in.

7. Two workmen, S and T, were engaged to work for a certain number of days. At the end of the time, S, who had idled 4 of these days, had 75*s*. to receive; but T, who had idled 7 days, received only 48*s*. Now, had T idled only 4 days, and S 7, they should have received exactly alike. For how many days were they engaged? *Ans.* 19.

Comparison of Similar Products and Surfaces.

The product of two numbers is said to be *similar* to the product of any two other numbers in the same ratio. Thus, the product of 4 × 5 is similar to that of 12 × 15.

In similar surfaces, all the lineal dimensions of one have the same interproportion as the corresponding lineal dimensions of another. Thus, in two similar quadrilateral figures, a diagonal and side of the one have to each other the same ratio as the corresponding diagonal and side of the other.*

Similar products of two factors are to each other as the squares of their corresponding factors. Or, the corresponding factors of similar products are to each other as the square roots of the products.

Similar surfaces are to each other as the squares of their corresponding lineal dimensions. Or, the corresponding lineal dimensions of similar surfaces are to each other as the square roots of the areas.

Examp. Two numbers are in the ratio of 16 : 7, and their product is $15\frac{3}{4}$. Required the numbers.

$16 \times 7 : 15{\cdot}75 :: 16^2 : 36$, the square of the factor corresponding to 16; therefore, that factor itself is 6; and the other factor is $\frac{7}{16}$ of 6, viz. $2\frac{1}{4}$. *Ans.*

Otherwise.

$\sqrt{112} : \sqrt{15{\cdot}75} :: 16 : 6$; where we may convert the first and second terms into the equivalent forms, $\sqrt{16 \times 7} : \sqrt{2{\cdot}25 \times 7} = 4 : 1{\cdot}5 = 8 : 3$.

Examp. If the diagonal of a room-floor is 12 feet, and its area is represented by 9; what is the corresponding area of a similar floor whose diagonal is 10 ft. 8 in.?

$$12^2 : (10\tfrac{2}{3})^2 :: 9 : 7\tfrac{1}{9} \text{ } \textit{Ans.}$$

Or, $12 : 10\frac{2}{3} :: \sqrt{9} : 2\frac{2}{3}$, the square root of the corresponding area; $2\frac{2}{3} \times 2\frac{2}{3} = \textit{Ans.}$

Examp. If the diameter of a circle be 9 inches; what is the diameter of a circle half as large?

* The diagonal and side of a square are incommensurable, and so are the diameter and circumference of a circle; but "we in many cases safely apply the conception of proportionality to examples in which one quantity is neither multiple, part, nor parts of another." Ol. Gregory's *Hints for Mathematical Teachers*, p. 34.

The proportion of areas is 2 : 1; therefore,

$$2 : 1 :: 9^2 : 40{\cdot}5;\ \sqrt{} = 6{\cdot}36396 \text{ in. } \textit{Ans.}$$

Or, $\sqrt{2} : \sqrt{1} :: 9 \text{ in.} : \textit{Ans.}$

Examp. Required two numbers such, that the greater may be $\frac{7}{12}$ of their sum, and that their sum multiplied by the less may be 270.

The less must be $\frac{5}{12}$ of their sum, and therefore the numbers are as 3 : 7. Then, the sum of these proportional parts multiplied by the less part is 30; consequently,

$30 : 270 :: 3^2 : 81\ ;\ \sqrt{} = 9$, the less number. *Ans.*

Or, $\sqrt{30} : \sqrt{270}$,

i. e. $\left.\begin{array}{rcl} \sqrt{1} : \sqrt{9} & :: & 3 : 9 \\ 3 : 7 & :: & 9 : 21 \end{array}\right\}$ *Ans.*

EXERCISES.

1. Find two numbers such, that the quotient of the greater divided by the less may be $3\frac{1}{2}$, and that thrice the greater multiplied by twice the less may give 650496. *Ans.* 616 and 176.

2. If the shortest side of an irregular rectilineal figure be 10; required the shortest side of a similar figure whose area is two-thirds of that of the former.

Ans. 8·164966.

3. If the side of a triangle measure 12 ft. 9 in., how much larger is a similar triangle of which the corresponding side measures 15 ft. 7 in.? *Ans.* $\frac{4}{9}$ths larger.

4. The diameter of a cylindrical vessel is $18\frac{1}{2}$ inches, and its depth 8 inches. What must be the diameter of one of the same capacity, whose depth is $7\frac{1}{2}$ inches?

Ans. 19·106718 in.

5. A maltster has a kiln, 14 feet in diagonal, but the area is too little by $\frac{1}{4}$ths for his business. Required the diagonal of a similar one which will answer his purpose. *Ans.* 18·78297 ft.

6. If a pipe of $1\frac{1}{4}$ inch bore can fill a cistern in 3·24

hours; in what time can the cistern be filled by a pipe of $3\frac{1}{2}$ inches bore? *Ans.* 38 h. 44 min.

7. The gilding of a globe, of 30 inches diameter, cost £2. 9*s*. Required the expense of gilding another, which is 43 inches in diameter. *Ans.* £5. 0*s*. $8\frac{1}{19}d$.

8. If the cost of planting a fence round a rectangular park be £2. 17*s*.; what should be the cost of fencing a similar space, which contains $2\frac{1}{2}$ square feet for every 5 of the former? *Ans.* £1. 18*s*. 11·26*d*.

9. There are three circles:—The area of the first is half that of the second, and the area of the third is 5 times that of the first. The circumference of the second circle is 13. Required the circumferences of the first and third. *Ans.* 9·1378 and 20·5548.

10. There is a rectangular field, whose breadth is $\frac{7}{15}$ of its length, and one-sixth of it being planted, there remain, for ploughing, 2730 square yards. Required the dimensions of the field. *Ans.* 78 yds. by 42.

11. Find two numbers, in the ratio of $\frac{1}{3}$ to $\frac{1}{4}$, the difference of whose squares shall be 425.

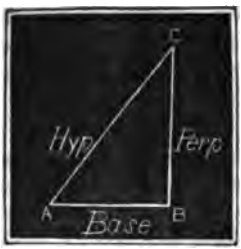
Ans. 40 and 45.

12. There are two square grass-plots, a side of the one being 4 yards longer than a side of the other; and their areas are as 4 : 3. What are the lengths of the sides? *Ans.* 29·856 and 25·856 yds.

13. There are two numbers, whose difference is $\frac{5}{13}$ of their sum, and the sum of their squares is 1000. Required the numbers. *Ans.* 29·24 and 12·04.

Comparison of the Sides of Right-Angled Triangles.

The sum of the squares of the *base* and *perpendicular* of a right-angled triangle is equal to the square of the *hypotenuse* or longest side: or, the difference between the square of the hypotenuse and that of the base or perpendicular is equal to the square of the remaining side.



Thus, the square of A C is the sum of the squares of A B and B C; or, the square of B C is the difference of the squares of A C and A B; or, the square of A B is the difference of the squares of A C and C B.

Note 1. Any three numbers in the proportion of 3, 4, 5, will constitute exact dimensions for the construction of a right-angled triangle.* Thus, if A B is 12 and B C 16, then A C is 20; for $12^2 + 16^2 = 400$, the square root of which is 20.

Note 2. Exact expressions for the same purpose may be found, by taking the sum of any two square numbers, their difference, and twice the product of their roots.† Thus, 64 + 25, 64 − 25, and 8 × 5 × 2, *i. e.* 89, 39, and 80, are sides of a right-angled triangle.

Note 3. The difference of the squares of two numbers may be determined expeditiously, by multiplying the sum of those numbers by their difference.‡ Thus, $23^2 - 21^2$, or 529 − 441, is equal to 44 × 2.

Examp. A wall is 28 feet high, and a ditch immediately along-side of it is 21 feet wide. What is the length of a ladder reaching the top of the wall from the outer edge of the ditch?

$$\sqrt{(28^2 + 21^2)} = \sqrt{1225} = 35 \text{ ft. } \textit{Ans.}$$

Examp. A ladder, 45 feet long, may be so placed on a street, that it will reach a window on one side, at the height of 43½ feet, and by turning it over, without shifting its position on the ground, it will reach a window on the other side, 27 feet high. Required the breadth of the street.

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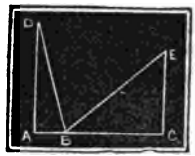
* This is because the square $25x^2$ is resolvable into the two squares $16x^2$ and $9x^2$.

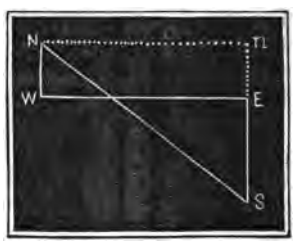
† This is because the square $x^4 + 2x^2y^2 + y^4$ is resolvable into the two squares $x^4 - 2x^2y^2 + y^4$ and $4x^2y^2$.

‡ This is because $x + y$ multiplied by $x - y$ gives $x^2 - y^2$.



A D is 43·2, C E 27, and B D or B E 45; therefore, $\sqrt{(45^2 - 43{\cdot}2^2)} = \sqrt{(88{\cdot}2 \times 1{\cdot}8)} = 12{\cdot}6$, A B. In like manner, $\sqrt{(45^2 - 27^2)} = \sqrt{(72 \times 18)} = 36$, B C. Accordingly, $36 + 12{\cdot}6 = 48{\cdot}6$ ft. A C. *Ans.*

Examp. One traveller sets out from E, and another from W, which is 80 miles west from E. The former travels 38 miles south, and the latter 22 miles north; how far are they then distant from each other?



Here, W E, or N *n*, is 80, and S *n* is 38+22, or 60;

$\sqrt{(80^2 + 60^2)} = 100$ mi. N S. *Ans.*

Examp. A tree, 63 feet high, growing on a horizontal plain, was *broken over* by the wind, and the top struck the ground at 21 feet from the root. How long was the piece that fell?

G T is the height of the tree, B the point at which it broke over, and R the point at which the top struck the ground. R B = B T; therefore, R B + B G = 63; and, as the sum of these, multiplied by their difference, gives R G^2 or 441, therefore, $441 \div 63 = 7 = \text{R B} - \text{B G}$. Consequently $(63 + 7) \div 2 = 35$ ft. R B or B T. *Ans.*

EXERCISES.

1. The base of a right-angled triangle is 266, and the perpendicular 312. Required the hypotenuse. *Ans.* 410.

2. A precipice rises perpendicularly 207 feet, and a river, flowing close past it, is of such width, that a line,

stretched from the farther edge of the water to the top of the precipice, measures 345 feet. Required the breadth of the river. *Ans.* 276 ft.

3. A line of 36 yards will exactly reach from the top of a fort to a station 24 yards from the bottom of the wall. Find the height of this wall.

Ans. 26·8328 yds.

4. There are two towers on a horizontal plain, the one 216 feet high, the other 135 feet. A rope stretched from the line of distance between them, at a point 180 feet from the bottom of the lower, will just reach the top of either. Required the length of the rope, and the distance between the towers. *Ans.* 225 ft; 243 ft.

5. Two travellers set out from London; one, due north, at the rate of 6 miles a day, the other, due east, at the rate of 8 miles a day. How far will they be distant from each other, 9 days after their departure?

Ans. 90 mi.

6. There is a tower, standing on the summit of a sloping hill whose ascent measures 160 feet; and the distance of the top of the tower from the foot of the hill is 213½ feet; also, the perpendicular height of the hill is 96 feet. Required the height of the tower.

Ans. 74⅛ ft.

7. There are two trees, 4 yards distant from each other, whose heights measure together 103 feet; and the top of the one is just 15 feet from that of the other. What is the height of each tree? *Ans.* 56 and 47 ft.

8. A ladder, when standing upright against a wall, exactly reaches its top; and, when its foot is drawn 18 feet from the bottom, its other end falls 6 feet from the top of the wall. Find the length of the ladder.

Ans. 30 ft.

9. A ladder may be so placed on a street 48·6 ft. wide, that it will reach a window on one side at the height of 43½ feet; and by turning it over, without shifting its position on the ground, it will reach a window on the other side, 27 feet high. Required the length of the ladder. *Ans.* 45 ft.

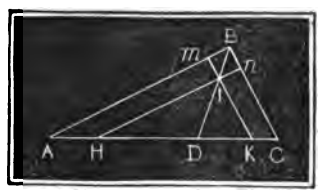
10. Two towers, M and N, whose heights are as 21 to 16, are so placed on a horizontal plain, that from the top of M to the top of N measures 175 feet, and from the top of N to the foot of M 180 feet. Find the height of each. *Ans.* M 58·21 ; N 44·35 ft.

Geometrical Problems.

*** The following Problems presuppose some acquaintance with Euclid's Elements. They are introduced here as involving applications of the Square Root, and affording a little practice in the adaptation of Arithmetic to Geometry.

Examp. There is a piece of ground in the form of a right-angled triangle, of which the base and perpendicular measure 20 and 44 yards, respectively; and along these sides runs a walk of uniform width, whose area is the same as that of the remaining triangle. Required the breadth of the walk.

Let A B C be the triangle, A B measuring 44 and B C 20. From B draw B D bisecting the angle A B C; and in B D find the point I, so that the square of D I may be half the square of D B,—which may be done by taking D I equal to one of the sides of an isosceles triangle inscribed in a semicircle upon B D. Through the point I draw K *m* and H *n* parallel to the sides C B and B A; then, *m n* is a parallelogram, having the alternate angles *m* B I and B I *n* equal; but the angles at B are made equal by bisection; therefore *m n* is a square, and I *m* = I *n*, which makes the breadth uniform throughout the walk A *n* K. Also the areas of the similar triangles A B C and A I K are to each other as D B^2 to D I^2, that is, as 2 to 1. Accordingly, the conditions of the Problem are fulfilled.

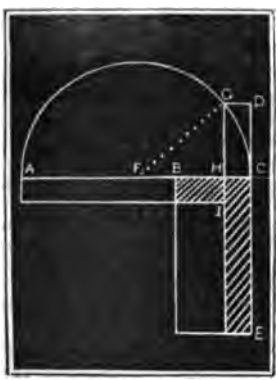


Arithmetically :—The area of A B C is half 44 × 20 ; therefore, the area of the walk is 220 ; and that area equals the product of the breadth of the walk by its mean length. Now, A B + B C = 64; and $\sqrt{2} : \sqrt{1} :: A B + B C : H I + I K$; therefore H I + I K=45·2548. Accordingly, the mean length of the walk is 54·6274, by which if we divide the area 220, we obtain 4·02728 yds. the width of the walk. *Ans.*

Examp. A rectangular looking-glass is surrounded by a frame

of uniform width, measuring at the outer edge 44 inches by 20; and the area of the frame is equal to that of the glass. Required the width of the frame.

Let the rectangle B E represent a quarter of glass and frame, E C measuring 22, and B C 10 inches. Produce C B to A, making A B = E C. On A C, from its middle point F, describe the semicircle A G C; and from C erect C D perpendicular to A C, and of such length that C D^2 may be equal to half the rectangle B E (Eucl. II. 14). Draw D G parallel to A C, and G H parallel to D C. Then, H C is the width of the frame; which being marked off from H to I, the quarter of the frame may be constructed.

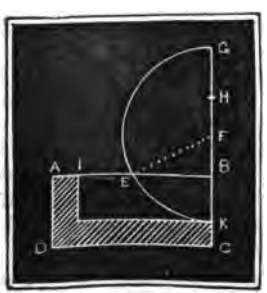


The construction is demonstrated by Eucl. VI. 8 and 17. Because G H is a mean proportional between A H and H C, therefore A H·H C = G H^2 = ¼ B E. Now A H·H C is evidently equivalent to A B·H C + B H·H C, or to E C·C H + B H·H I. Wherefore, half the rectangle B E is equal to the quarter frame E H + B I.

Arithmetically: F G^2 = 16 × 16; G H^2 = 11 × 10; diff. F H^2 = 146. Therefore, F H = $\sqrt{146}$ = 12·083, which subtracted from F C, 16, leaves H C = 3·917 in. width of frame. *Ans.*

Otherwise.

Let A B C D represent a quarter of glass and frame; A B 22, B C 10 inches. From B A cut off B E, the side of a square equal in area to half the rectangle A B C D. Produce C B to H, making B H = A B − B C; and from F, the middle point of B H, with the radius F E, describe the semicircle G E K. Then, C K is the breadth of the frame; which being marked off from A to I, the quarter of the frame may be constructed.

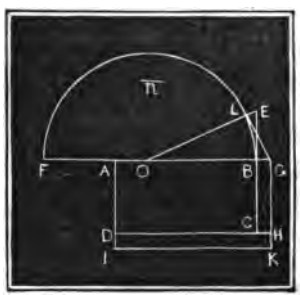


For demonstration of this construction, we have C H = A B, and G H = B K, therefore C K + B G = A B, or B G = B I; also, B G·B K = B E², or B I·B K = B E² = ½ A B C D.

Arithmetically: B F = ½ (22 − 10) = 6, or B F² = 36, and B E² = 11 × 10; sum, E F² = 146. Therefore, E F = F K = 12·083, which subtracted from F C, 16, leaves K C = 3·917 in. width of frame. *Ans.*

Examp. A gentleman has a bowling-green, 230 feet long and 120 feet broad, which he wishes to raise one foot higher by means of the earth dug out of a trench surrounding it. Required what must be the uniform width of the trench, if its depth be every where 4 feet 2 inches.

Let A B C D represent one-fourth of the green, A B being 115, and B C 60 feet. Now, dividing A B × B C (the quantity of earth taken out of a fourth of the trench) by the given depth of the trench, finds the area of one-fourth of it. Therefore, this area may be represented by the rectangle A B × *n*, that is 115 × 14·4.



Produce B A to F, making A F = A D, and bisect F B in O. Upon F B describe a semicircle, and erect B E perpendicular to A B, so that B E² may equal A B × *n*. Join O E, and through the point L draw the tangent L G meeting A B produced in G; then B G is the width of the trench; and, consequently, a fourth of its area may be represented by B H I.

The construction is demonstrated by Eucl. I. 26, and III. 36; for, by the former of these Propositions, we have G L = E B, and by the latter, F G·G B = G L² = B E² = the area of one-fourth of the trench. But F G·G B is = A G·G B + F A·G B = D H·H K + H G·G B; therefore, B H I is one-fourth of the trench.

Arithmetically: O L² + L G² = 87·5² + 115 × 14·4 = 9312·25, the square root of which is 96·5 = O G. Hence O G − O B = B G = 9 feet. *Ans.*

EXERCISES.

1. The plate of a rectangular looking-glass measures 34 inches by 16, and it is surrounded by a frame of such uniform width, that the area of the frame is equal to that of the glass. Required

the width of the frame, by a demonstrated geometrical construction, and also by arithmetical computation.

Ans. 4·59532 in.

2. A gentleman has a bowling-green, 300 feet long and 200 feet broad. He proposes to diminish its area, and raise the surface 1½ foot higher, by means of a trench dug round it, within the perimeter. Required, geometrically and numerically, what must be the uniform width of the trench, if the depth be everywhere 9½ feet. *Ans.* 8·715 ft.

3. A rectangular field, 258 feet wide, is contiguous to a square field of the same area, and both fields together measure 555 feet in length. Required the width of the square field, by geometry and arithmetic. *Ans.* 270·7887 ft.

4. There are four trees, A, B, C, D: A is 660 yards west from B; C is in a direct line between A and B, and exactly south from D, which is subtended by A and B at a right angle. Also, the distance from A to C is equal to the distance from B to D. Determine geometrically the positions of the trees, and, numerically, the distances between them. *Ans.* A C 407·9; C D 320·668; &c.

APPLICATIONS OF THE CUBE ROOT.

Comparison of Similar Products and Solids.

The product of three numbers is said to be *similar* to the product of any three other numbers in the same proportion. Thus, the product of $3 \times 5 \times 9$ is similar to the product of $15 \times 25 \times 45$.

Similar products, by three factors, are to each other as the cubes of their corresponding factors. Or, the corresponding factors are as the cube roots of the products.

The volumes of similar solids, and the weights of similar solids of the same density, are to one another as the cubes of their corresponding lineal dimensions; or, these dimensions are as the cube roots of the volumes or weights.

Examp. If 2½ lbs. of gunpowder be sufficient to charge a cannon of 3 inches bore, what quantity will be requisite to charge a cannon of 8 inches bore?

$3^3 : 8^3 :: 2\frac{1}{2}$ lbs. $: 47\frac{1}{17}$ lbs. *Ans.*

Or, $3 : 8 :: \sqrt[3]{2 \cdot 5} : \sqrt[3]{2 \cdot 5 \times \frac{1}{2}}$; which being cubed gives $(8^3 \times 2 \cdot 5) \div 3^3$, the same as before.

Examp. The diameter of an iron ball is 6 inches; required the diameter of a leaden ball one-third as heavy; the specific gravity of the iron being 7207, and that of the lead 11352.

$$\left.\begin{matrix} 3 : 1 \\ 11352 : 7207 \end{matrix}\right\} :: 6^3 : 45 \cdot 7103594$$

$\sqrt[3]{45 \cdot 7103594} = 3 \cdot 5755$ inches. *Ans.*

Examp. If 27 grains of gold gild a ball weighing 729 ounces, how many will gild one of the same material that weighs 1728 ounces?

The diameters are as $\sqrt[3]{729} : \sqrt[3]{1728}$, or as 9 : 12, or as 3 : 4; then, the surfaces being as the squares of the diameters, we have

$$3^2 : 4^2 :: 27 \text{ grs.} : 48 \text{ grs.} \quad \textit{Ans.}$$

Examp. Required three numbers proportioned as 3, 7, and 9, so that the cube of the third may exceed the sum of the cubes of the two first by 123137.

$$9^3 = 729$$
$$3^3 + 7^3 = 370$$

$$359 : 123137 :: 729 : 729 \times 343,$$

the cube root of which is 9×7, or 63, the third number, so that the numbers are 21, 49, and 63. *Ans.*

EXERCISES.

1. Required the side of a vessel, the content whereof shall be twice that of a similar vessel whose side is 9 inches. *Ans.* 11·33929 in.

2. If a cannon ball of 3 inches diameter weigh $3\frac{1}{2}$ lbs., what is the weight of a ball whose diameter is $5\frac{1}{2}$ inches. *Ans.* $21\frac{1}{4}\frac{1}{4}\frac{1}{4}$ lbs.

3. How many times as heavy is a leaden ball of 8 inches diameter, compared with one of $3\frac{1}{2}$ inches diameter? *Ans.* $11\frac{1}{4}\frac{1}{4}\frac{1}{4}$ ti.

4. A log of mahogany, 12 feet long, 3 broad, and 2¼ thick, weighs 35 cwt. Find the dimensions of a similar log weighing 1 ton.

Ans. 9·958, 2·489, and 2·282 ft.

5. Required the dimensions of a block of limestone, half as heavy as a similar block of granite 20 inches long, 15 broad, and 8 thick; the specific gravity of the limestone being 2950, and of the granite 3500.

Ans. 16·804, 12·603, and 6·721 in.

6. If 8*s.* 6*d.* be charged for painting a block of wood, whose cost, computed at so much per cubic foot, is 83*s.* 4*d.*, what is the corresponding cost of a similar block, the charge for painting which, in the same style, is 18*s.* 4*d.*? *Ans.* £13. 3*s.* 11·56*d.*

7. What two numbers are as 5 : 4, the sum of their cubes being 5103? *Ans.* 15 and 12.

8. Find three numbers such, that 11 times the first may equal 5 times the second, and 8 times the second equal 9 times the third; the continual product of the numbers being 10585080. *Ans.* 135, 297, and 264.

Kepler's Third Law.

Newton demonstrated the law ascertained by Kepler, that the cubes of the mean distances of the planets from the Sun are as the squares of their times of revolution.

Examp. If the Earth's mean distance from the Sun be 95071447 miles, and its sidereal year 365 da. 6 ho. 9 min. 10 sec., required the distance of the planet Pallas, reckoning its sidereal year to be 1686 da. 12 ho. 55 min. 50 sec.

Proportion of Periods = 1 : 4·617411033, the cube root of the latter being 1·6651991; and the square of that root shows the distance of Pallas to be 2·772888 times that of the Earth, or 263622474 miles. *Ans.*

Examp. The planet Venus is distant from the Sun 68768256 miles, and Jupiter 494482330 miles. Re-

quired Jupiter's period of revolution; that of Venus being 224 da. 16 ho. 49 min. 11 sec.

Jupiter's distance is 7·19056086 times that of Venus; and the cube of the square root of that number, or, which is the same thing, the number multiplied by its square root, 2·6815221, gives 19·2816478; by which the time of Venus being multiplied, we have Jupiter's sidereal year, 4332 da. 14 ho. 27 min. 2 sec. *Ans.*

EXERCISES.

1. What is the proportion of the periods of revolution of the Earth and Mars, their mean distances being, respectively, 95071447 and 144859632 miles?

Ans. 1 to 1·880814.

2. The sidereal year of the Georgium Sidus is 30737 da. 17 ho. 59 min. 50 sec., and that of Mercury is 87 da. 23 ho. 15 min. 36 sec. What is the mean distance of Mercury from the Sun, that of the Georgium Sidus being 1825714895 miles? *Ans.* 36801967 mi.

3. If the Earth's mean distance from the Sun be 95071447 miles, and its sidereal year 365 da. 6 ho. 9 min. 10 sec., what is the sidereal year of Saturn, whose distance is 906787297 miles? and what is the time of a mean synodic revolution* of that planet with the Earth?

Ans. Sid. year 10759 da. 5 ho. 16 min. 8 sec.
Syn. Rev. 378 da. 2 ho. 12 min. 21 sec.

POSITION.

POSITION, called also False Position, or False Supposition, is a Rule which, by means of Trial and Error, obtains the true answer to a question.

Problems in *Single Position* are such as can be

* "The synodic revolution of a planet is the time of its returning from any position in respect to the earth into that position again." See Moseley's *Lectures on Astronomy*, Ch. 62.

solved by *one* trial assumption; those in *Double Position* require *two*.

Single Position.

Single Position is applicable for the solution of certain indirect problems, in which the required number is to be increased or diminished by some multiple or some proportional part of itself.

Assuming any number for that required, if we operate upon it as the question directs, we shall obtain a result bearing the same proportion to the result specified in the question, as the assumed number bears to the one required.

Example. Find a sum of money, from which its third and its seventh being subtracted, there shall remain £16. 10*s*.

Suppose the sum to be £42, the third of which is £14, and the seventh £6; then $42 - 14 - 6 = £22$, should be £16. 10*s*.; therefore,

$$22 : 16\tfrac{1}{2} :: £42 : £31.\ 10s.\ \textit{Ans.}$$

The reason of this Rule is obvious; for when ratio is the only kind of relation admitted among the terms of a question, all the steps of the procedure issuing from a supposed number, must bear a common ratio to the corresponding results of similar operation on the required number.

Problems suitable for Single Position are in general very easily resolvable with less arbitrary assumption; and, therefore, this Rule merits little distinction.

In both Single and Double Position, it is expedient, though not necessary, to assume such numbers as will keep the process free from fractions.

EXERCISES.

1. A father's age is 4 times that of his son; and if a seventh of the difference of their ages be added to $2\frac{3}{4}$, the sum will be an eighth of the sum of their ages. Find the age of each. *Ans.* 56 and 14.

2. What sum improved at 4 per cent. per annum, compound interest, will gain £30 in 3 years?

Ans. £240. 5*s*. $2\frac{1}{4}d$.

3. A number is increased by a third of its double; and the amount being then diminished by 320, leaves 1 less than a twelfth of the previous increase. Required the number. *Ans.* 198.

4. What two numbers, in the proportion of 13 to 14, are such, that the sum of 10 times the less and 7 times the greater exceeds 3 times the difference of the numbers by 75? *Ans.* $4\frac{1}{2}$, $4\frac{8}{13}$.

Double Position.

By Double Position may be resolved all questions to which Single Position is applicable; but its special application is to certain indirect problems containing the addition or subtraction of quantities not depending on the number sought, *i. e.* having no assigned ratio to it.

When the terms of a question, by their algebraic arrangement, form a simple equation, it may be resolved by Double Position. Thus, if we assume any two convenient numbers, and operate with each of them as the question directs to do with the required number, each supposition, if false, will of course result in an error of excess or of defect; and it is demonstrable that the difference of the errors is to the difference of the assumptions, as either error of result is to the error of the supposition which produced that result. So that, by this analogy of errors the correct answer will be found.*

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* The truth of this analogy may be algebraically investigated as follows:—Any simple equation may be represented by the form $ax = c$, where a and c may each denote the collected result of several given quantities related by $+$ or $-$.

Now, if $ax = e$, then $ax \backsim c = 0$.

Let f be any falsely supposed value of x, and let e be the error of the result; also, let F be any other falsely supposed value of x, and E the error of the result.

Then, instead of $ax \backsim c = 0$, we have $af \backsim c = e$, and $aF \backsim c = E$; and, by substituting for c its equivalent ax, we have $af \backsim ax = e$, and $aF \backsim ax = E$. Dividing the latter equation by the former, we obtain

$$\frac{F \backsim x}{f \backsim x} = \frac{E}{e} \text{ or } E : e :: F \backsim x : f \backsim x;$$

therefore, $E \backsim e : F \backsim f :: e : f \backsim x :: E : F \backsim x$.

The errors of the results should be marked + or −, according as they denote excess or defect; and when one of the errors is + and the other −, it must be remembered that the difference of such errors is the sum of their numbers; thus, between 4 of excess and 3 of deficiency there is a difference of 7.

Examples.

1. A man leaves to his eldest child one-fourth of his property; to his second, one-fourth of the remainder and £350 besides; to his third, one-fourth of the last remainder and £199 besides; and what still remains he bequeaths to his wife, whose share is found to be one-fifth of the whole. Required the value of the whole property.

Suppose the whole to be	£8800	£7200
Eldest child gets	2200	1800
	4)6600	4)5400
	1650	1350
	350	350
Second child	2000	1700
	4)4600	4)3700
	1150	925
	199	199
Third child	1349	1124
Widow's share	3251	2576
Should be one-fifth, or	1760	1440
Errors	+1491	+1136

+1491 8800
+1136 7200

355 : 1600 : : 1136 : 5120, error of 2nd supposition; therefore, $7200 - 5120_2 = £2080$. *Ans.*

2. A farmer sold at one time 30 bushels of wheat and 40 of barley, and for the whole received £13. 10*s*., and at another time he sold, at the same prices as before, 50 bushels of wheat and 28 of barley, and for the whole received £16. 14*s*. Required the selling prices per bushel.

Suppose:

30 bu. W. at 9*s*.	= 270*s*.	30 bu. W. at 0*s*.	
40 bu. B. at 0*s*.		40 bu. B. at 6*s*. 9*d*.	= 270*s*.
50 bu. W. at 9*s*.	= 450*s*.	28 bu. B. at 6*s*. 9*d*.	= 189*s*.
Should be	334*s*.	Should be	334*s*.
Errors	+116*s*.		−145*s*.

+116	9
−145	0

261 : 9 :: 145 : 5*s*. error of 2nd supposed price of wheat; therefore, wheat sold at 5*s*. and barley at 3*s*. *Ans*.

3. A company at a tavern found that the amount of reckoning would have been 1*s*. 6*d*. more from each person, had there been 3 fewer in the party, or 1*s*. 6*d*. less from each, had there been 6 more in the party. Required the number of persons and the quota of each.

Supp. the no. of persons to be	8		10
3 fewer (5) at 1*s*. 6*d*.	7*s*. 6*d*.	(7)	10*s*. 6*d*.
÷ 3 gives the quota	2*s*. 6*d*.		3*s*. 6*d*.
Therefore the amount is	20*s*.		35*s*.
6 more (14) at 1*s*. 6*d*. less is	14	(16)	32
Errors	− 6		− 3

6 − 3 : 10 − 8 :: 6 : 4 the correction for supp. 8; Therefore, there were 12 persons, at 4*s*. 6*d*. *Ans*.

EXERCISES.

1. After A had won 4*s*. of B, he had only half as many shillings as B had left; but, had B won 6*s*. of

A, he would have had thrice A's remaining number. How many shillings had each? *Ans.* A 36*s.*, B 84*s.*

2. A certain number is multiplied by 4, and the product diminished by 20; the remainder being then divided by 7, and 15 added to the quotient, the result is just the original number. Find that number.

Ans. 28½.

3. A son having asked his father's age, the latter replied, "If you take 5 from my years, and divide the remainder by 8, the quotient is a third of your age; but if 2 be added to your age, thrice the sum, diminished by 7, gives my age." Required the age of each.

Ans. 53, 18.

4. Divide the number 90 into four parts, so that the first + 5, the second − 4, the third × 3, and the fourth ÷ 2, may be equal quantities.

Ans. 16, 25, 7, 42.

5. A and B speculate in trade with different sums. A gains £60, and B loses £20, and now A's stock is to B's as 3 to 2. But had A lost £170 and B gained £110, A's stock would have been to B's as 5 to 9. What was the original stock of each? *Ans.* A £420, B £340.

6. Find two numbers in the proportion of 4 to 5, from which two other required numbers, in the proportion of 6 to 7, being, respectively, deducted, the remainders shall be in the proportion of 2 to 3, and their sum 20. *Ans.* 32 and 40, 24 and 28.

7. A lady being asked how many yards of silk she had in her gown, replied that if it contained 2 yards more, and the price had been 3*s.* a yard more, the gown would have cost 44*s.* 8*d.* more than it did; but if it contained 3 yards more, and had cost 2*s.* a yard more, she must have paid for it 41*s.* 6*d.* more than she did. Required the number of yards in the gown, and the price per yard. *Ans.* 9 yds. at 5*s.* 10*d.*

8. A rectangular green having been measured, it was observed that if it were 5 feet broader and 4 feet longer, it would contain 116 square feet more; but if it were

4 feet broader and 5 longer, it would contain 113 feet more. Required the length and breadth.

Ans. 12 ft. by 9 ft.

Note.—Bonnycastle, who says that to the Arabs we are indebted for the Rules of Position, devised "a much more simple and commodious rule for Double Position" than any which had previously been given. His method, however, which was first published in the 8vo. edition of his Arithmetic (1810), is not sufficiently general in its application; for, instead of prescribing the difference of the *errors* of result as the first term of analogy, he directs to take the difference of the false *results* themselves.

Our second example would admit the application of his method, because the required result, 334, is *absolute;* but it suits neither the first nor the third example, because in each of these the required result is *proportionally related* to some foregoing quantity.

All problems which Position can resolve exactly, may be wrought, though in many instances not so easily, by some of the other Rules of Arithmetic; and, as questions in this Rule are generally of no practical use in themselves, the facility it may afford in solving a problem cannot be regarded as compensating the diminution of that intellectual exertion which a more direct mode of solution would demand. The Rule, however, is entertaining, and deserves to be known; and any teacher who may think the above Exercises too few, can easily extend their number by appropriating some of the Miscellaneous Questions in this volume.

Approximating Position.

Double Position is a useful expedient for the solution of many questions which constitute algebraic equations of the higher degrees, and for which only approximate answers can be found. "When the proposed equation involves the higher powers of x, then the errors" ($F \backsim x$ and $f \backsim x$) "must occur in the higher powers; so that the rule, by supposing these higher powers absent, either falls short of the truth or exceeds it—by a quantity, however, which becomes more and more unimportant, as the original errors become diminished" (*Young's Algebra*). Accordingly, after we have tried two values, as near the truth as we can guess, and to which the truth is intermediate, the answer will suggest new trial numbers, whereby a still nearer approximation to accuracy may be attained.

Example.

A certain number, the square of that number, and the cube of it, are aggregate parts of unity. Required the number.

It appears on trial that the required number is greater than ·5, and less than ·6. Let us, therefore, employ these assumptions.

If the number is	·5	·6
its square is	·25	·36
and its cube is	·125	·216
	·875	1·176
Should be	1·000	1·000
Errors	− ·125	+ ·176

·301 : ·1 : : ·125 : ·04 the error of ·5; therefore, the required number is, nearly, ·54.

Let us now make trial of ·54 and ·55.

If the number is	·54	·55
its square is	·2916	·3025
its cube is	·15746	·16637
	·98906	1·01887
Should be	1·00	1·00
Errors	− ·01094	+ ·01887

·02981 : ·01 : : ·01094 : ·0036 error of ·54; therefore, the number sought is, more nearly, ·5436.

By making our next trial with ·5436 and ·5437, we should obtain the very near approximation ·54368901, which is true to the last figure. *Ans.*

EXERCISES.

9. What number is 1 less than its third power?

Ans. 1·324718.

10. Required a number, to the cube of which 12 times the number being added, the sum is 300. *Ans.* 6·098535.

11. What number, added to its second, third, and fourth powers, amounts to 41? *Ans.* 2·1981808.

12. Required the 5th root of 1·03. *Ans.* 1·00592927.

13. Required the 7th root of 56. *Ans.* 1·7772198.

14. What number raised to its 13th power, gives the 4th root of 1·03? *Ans.* 1·0005686.

EQUIDIFFERENT SERIES.

A series of numbers increasing or decreasing by a common difference, is called an EQUIDIFFERENT SERIES, and sometimes, though improperly, an Arithmetical Progression. Thus, in the series 5, 8, 11, 14, 17, &c., or 17, 14, 11, 8, 5, &c., the common difference is 3.

It is obvious from the nature of such a series, that the sum of any two extreme terms equals the sum of any two intermediate terms equidistant from those extremes, and equals twice the mean term. Thus, $5 + 17 = 8 + 14 =$ twice 11.

It is also readily perceived, that the product of the common difference by 1 less than the number of terms, is the amount of increase from the least to the greatest term. Thus, $5 + 3 \times 4 = 17$.

The mean of any number of terms of an equidifferent series, multiplied by the number of terms, will manifestly produce the sum of the series; thus, $5 + 8 + 11 + 14 + 17 =$ 5 times 11; and the mean term being half the sum of the extremes, we observe that the sum of an equidifferent series equals the number of terms multiplied by half the sum of the extremes.

Representing, therefore, the greatest term, least term, common difference, number of terms, and sum of the series, respectively, by the initial letters g, l, d, n, s, we have found

$$g = l + d \times (n - 1);$$
$$s = \tfrac{1}{2}(g + l) \times n.$$

By means of these equations the following Theorems are constructed; and it is presumed that the student is sufficiently acquainted with the signs to be able to translate the formulae for the resolution of problems.

1. Given l, n, d, to find g and s.
   $g = l + d \times (n - 1)$; $s = [\frac{1}{2} d \times (n - 1) + l] \times n$.
2. Given g, d, n, to find l and s.
   $l = g - d \times (n - 1)$; $s = [g - \frac{1}{2} d \times (n - 1)] \times n$.
3. Given g, l, d, to find n and s.
   $n = (g + d - l) \div d$; $s = \frac{1}{2}(g + l) \times (d + g - l) \div d$.

4. Given g, l, n, to find d and s.
$$d = (g - l) \div (n - 1);\ s = \tfrac{1}{2}(g + l) \times n.$$

5. Given l, n, s, to find g and d.
$$g = 2s \div n - l;\ d = 2(s - ln) \div (n^2 - n).$$

6. Given g, n, s, to find l and d.
$$l = 2s \div n - g;\ d = (gn - s) \div \tfrac{1}{2}(n^2 - n).$$

7. Given g, l, s, to find n and d.
$$n = 2s \div (g + l);\ d = \frac{(g + l) \times (g - l)}{2s - g - l}.$$

8. Given d, n, s, to find l and g.
$$l = s \div n - \tfrac{1}{2}d \times (n - 1);\ g = s \div n + \tfrac{1}{2}d \times (n - 1).$$

9. Given g, d, s, to find l and n.
$$l = \frac{\sqrt{(2g + d)^2 - 8ds} + d}{2};\ n = \frac{2g + d - \sqrt{(2g + d)^2 - 8ds}}{2d}$$

10. Given l, d, s, to find g and n.
$$g = \frac{\sqrt{(2l - d)^2 + 8ds} - d}{2};\ n = \frac{\sqrt{(2l - d)^2 + 8ds} - 2l + d}{2d}$$

Examp. Find the 25th term of the series 4, 9, 14, &c. Here $l = 4$, $n = 25$, $d = 5$; to find g.

By 1st Theorem, $g = 4 + 5 \times 24 = 124$. *Ans.*

Examp. Given the greater extreme 500, the less 70, and the common difference 10; to find the number of terms, and the sum of the series.

Here $g = 500$, $l = 70$, $d = 10$; to find n and s.

By 3rd Theor. $n = (500 + 10 - 70) \div 10 = 44$. *Ans.*

$s = \frac{1}{2}(500 + 70) \times (500 + 10 - 70) \div 10 = 12540$. *Ans.*

Examp. How many times does a common clock strike in a year?

Here $l = 1$, $d = 1$, $n = 12$; to find s, the number of strokes in the 12 hours.

Or, $l = 1$, $g = 12$, $n = 12$; to find s.

By 1st. Theor. $[\frac{1}{2}(12 - 1) + 1] \times 12 = 78$;

Or, by 4th Theor. $\frac{1}{2}(12 + 1) \times 12 = 78$;

$78 \times 730 = 56940$ times in a year. *Ans.*

Examp. If a man travel 10 miles the first day, 14 the second, 18 the third, and so forth; in what time

will he travel as far as if his journeys had been uniformly 40 miles a day?

40 being the mean term, the sum of the extremes must be $40 \times 2 = 80$; therefore, $80 - 10 = 70$, the greater extreme. Given $g = 70$, $l = 10$, $d = 4$; to find n.

By 3rd Theor. $n = (70 + 4 - 10) \div 4 = 16$ da. *Ans.*

Examp. Y saves from his salary £38 the first year, £34 the second, £30 the third, and so on; also, Z, from his income, lays by £11 the first year, £13 the second, £15 the third, and so forth. In a certain time, the amount of Z's savings equals the amount of Y's. Required that time.

Here, it is observed, that Y gains on Z £27 the first year, £21 the second, &c., decreasing yearly by $4 + 2$ or £6; and as $27 \div 6 = 4$, with 3 remainder, Y's successive gains upon Z terminate with 3 as the 5th term, and then Z's successive gains upon Y begin with 3, and increase in each term by 6; but this latter portion of the series will just be an inverted repetition of the former, and, accordingly, the amount of Z's savings will equal that of Y's in $5 + 5 = 10$ years. *Ans.*

Examp. Suppose that for 14 days a man travels on each succeeding day $1\frac{1}{2}$ mile less than he did the preceding, and that during the first 5 days he travels altogether $65\frac{1}{2}$ miles less than in the remaining 9 days; what is the whole length of his journey?

$1\frac{1}{2} \times 5$ or $7\frac{1}{2}$ miles is the difference between each of the first 5 days and each of the second 5, respectively; hence, in the second 5 days, he travels $7\frac{1}{2} \times 5$ or $37\frac{1}{2}$ miles less than during the first 5; and therefore in the last 4 days he travels $37\frac{1}{2}$ miles and $65\frac{1}{2}$ miles more. Given, therefore, the sum of 4 terms, 103, and the common difference $1\frac{1}{2}$, to find the least term. This, by 8th Theor. is $23\frac{1}{2}$ miles travelled on the 14th day.

Accordingly, for the whole journey, we have $l = 23\frac{1}{2}$, $n = 14$, $d = 1\frac{1}{2}$; to find s. And this, by 1st Theor. is found to be $465\frac{1}{2}$ miles. *Ans.*

When an equidifferent series is extended beyond an integral number of terms, to include a fractional approximation to the next term, the assigned fraction denotes the addition of a proportional part of the common difference; as in the annexed example, which finds the sum of $5\frac{3}{4}$ terms of the series 5, 9, 13, &c.

1st term,		= 5
2nd,	5 + 4	= 9
3rd,	9 + 4	= 13
4th,	13 + 4	= 17
5th,	17 + 4	= 21
$5\frac{3}{4}$th,	21 + $\frac{3}{4}$ of 4	= 24
	Sum,	89

Now, when the last number of a series is thus imperfect, the value of s is not directly provided for by the Theorems which we have furnished. And, when s is the given sum of a broken series, n must also be given, or the three elements will not suffice to determine the problem.

Examp. In an increasing equidifferent series the least term is 5, the number of terms $5\frac{3}{4}$, and the common difference 4. Required the last or broken term, and the sum of the series.

By 1st Theor. $g = 5 + 4 \times 4\frac{3}{4} = 24$; *Ans.*
Sum of 5 terms $= (2 \times 4 + 5) \times 5 = 65$.
Sum of $5\frac{3}{4}$ terms $= 65 + 24 = 89$. *Ans.*

Examp. In a broken increasing series, given $l = 5$, $g = 24$, $d = 4$; to find n and s.

By 3rd Theor. $n = (24 + 4 - 5) \div 4 = 5\frac{3}{4}$ terms. *Ans.*

By 1st Theor. the sum of 5 terms is found to be 65; and $65 + 24 = 89$, the sum of $5\frac{3}{4}$ terms. *Ans.*

Examp. In a broken increasing series, given $l = 5$, $n = 5\frac{3}{4}$, $s = 89$; to find d and g.

By 1st Theor.

The $5\frac{3}{4}$th term is	$= 4\frac{3}{4}d + 5$
Sum of 5 terms	$= 10\ d + 25$
Sum of $5\frac{3}{4}$ terms	$= 14\frac{3}{4}d + 30$

$(89 - 30) \div 14\frac{3}{4} = 4$, the com. dif.
$4\frac{3}{4} \times 4 + 5 = 24$, greatest term } *Ans.*

Examp. In a broken decreasing series, the least or imperfect term is 6, the number of terms $5\frac{3}{4}$, and the sum 91. Find the common difference.

The 5th term is $6 + \frac{5}{4}d$; therefore, by 1st Theor. the sum of 5 terms $= 30 + 13\frac{1}{4}d$; but that sum is $91 - 6 = 85$; therefore, $(85 - 30) \div 13\frac{1}{4} = 4$, the com. dif. *Ans.*

Examp. In a broken increasing series, given $d = 3$, $n = 6\frac{1}{2}$, $s = 95\frac{1}{2}$; to find g.

By 1st Theor. the $6\frac{1}{2}$th term is $l + 15\frac{1}{2}$; and the sum of 6 terms is $6l + 45$; so that the sum of $6\frac{1}{2}$ terms is $7l + 60\frac{1}{2}$; wherefore, $(95\frac{1}{2} - 60\frac{1}{2}) \div 7 = 5$, the least term, and $6l + 45$, or 75, is the sum of 6 terms; $95\frac{1}{2} - 75 = 20\frac{1}{2}$. *Ans.*

EXERCISES.

1. Given the greatest term of an equidifferent series, 55, the common difference 2, and the number of terms 26; to find the least term, and the sum of the series. *Ans.* 5; 780.

2. The greatest term being $160\frac{1}{2}$, the least term $7\frac{1}{2}$, and the common difference $8\frac{1}{2}$; required the number of terms, and the sum of the series. *Ans.* 19; 1596.

3. Given the extremes $5\frac{1}{8}$ and $7\frac{1}{8}$, and the number of terms 14; to find the uniform increase, and the sum of the series. *Ans.* $\frac{5}{7\,7}$; $90\frac{1}{2}\frac{7}{7}$.

4. A person travels from A to B, 406 miles, in 14 days. He goes 9 miles the first day, and increases each day's journey by an equal excess. Required that excess. *Ans.* $3\frac{1}{1\,3}$ mi.

5. If £124. 7*s.* 6*d.* were divided among 100 men, the shares rising above each other by equal excesses, and the first share being $1\frac{1}{2}d.$; what would the last share be? *Ans.* 49*s.* $7\frac{1}{2}d.$

6. Into how many equidifferent shares may 5*s.* $11\frac{1}{2}d.$ be divided, so that the greatest share may be $8\frac{1}{2}d.$, and the common difference $\frac{1}{2}d.$? *Ans.* 13.

7. Suppose that from a cask, containing 500 gallons, there are drawn off 11 successive quantities, forming an

equidifferent series, of which the greatest is 74 gallons; and that 71 gallons are then left in the cask; what will the smallest of those quantities be? *Ans.* 4 gall.

8. Suppose 400 stones placed in a straight line, at intervals of a yard from each other, and a basket placed a yard from the first; how far must a man travel in bringing them singly to the basket, if he begin at the last stone? *Ans.* 90 mi. 7 fur. 60 yds.

9. A body falling by its own weight, if it were not resisted by the air, would descend in the first second of time through a space of 16 ft. 1 in., in the next second through 3 times that space, in the next through 5 times that space, in the next through 7 times, &c. Through what space would it fall, at the same rate of increase, in a minute? *Ans.* 57900 ft.

10. Suppose a steam carriage runs the first second at the rate of a mile per minute, but in the next second abates the 20th part of its speed, in the next the 19th part, in the next the 18th part, &c. How far will it run altogether? *Ans.* 924 ft.

11. A merchant, who was engaged in trade 18 years, gained the first year £37; and his total gain, abating $\frac{4}{17}$ of the last year's, was £1093. Required how much he gained the second year, upon supposition that his gains advanced by an equal increase yearly. *Ans.* £40.

12. A and B set out together from the same position, and travel in the same direction; A goes uniformly 18 miles a day; B goes 5 miles the first day, 7 the second, 9 the third, and so on. In what time will B come up with A? *Ans.* 14 da.

13. A and B depart together from the same place, and travel in the same direction. A goes 74 miles the first day, 78 the second, 82 the third, and so on; B goes 14 miles the first day, 23 the second, 32 the third, &c. How long and how far must they travel at these rates, to let B overtake A? *Ans.* 3050 mi. in 25 da.

14. Divide 1414 guineas into 7 shares forming an

equidifferent progression, of which the last term shall be 100 times the first. *Ans.* 4, 70, 136, &c. guineas.

15. In a broken increasing series of equidifferent sums of money, the extremes are 10*s.* and 14*s.* 11*d.*, and the number of terms is $20\frac{3}{4}$. Required the common difference. *Ans.* 3*d.*

16. Given 121 as the greatest term in a broken ascending series, whose number of terms is $12\frac{5}{7}$, and sum 889; to find the least term and the common difference. *Ans.* $14\frac{1}{2}$; 9.

17. Given 1 as the greatest term in a descending broken series of which the number of terms is $13\frac{1}{3}$, and the sum 8·57; to find the least term and the common difference. *Ans.* $\frac{1}{4}$; ·06.

18. M and N set out from a certain place at the same time, and perform a certain journey in the same direction in $11\frac{1}{2}$ days. M travels at the uniform rate of 26 miles per day; while N commences at the rate of 8 miles a-day, and increases each following day's journey by a common difference. Required that common difference. *Ans.* $3\frac{4}{23}\frac{8}{3}$ mi.

EQUIRATIONAL SERIES.

When a series of numbers increases or decreases by a common multiplier or divisor, that is, by a common Ratio, it is called an EQUIRATIONAL SERIES, or a series of Continual Proportionals, and very often, though improperly, a Geometrical Progression. In such a series the first term bears the same ratio to the second, as the second to the third, the third to the fourth, &c. Thus, 3, 6, 12, 24, 48, &c., and 54, 18, 6, 2, $\frac{2}{3}$, &c., are series of Continual Proportionals, the common ratio in the former being 2, and in the latter 3.

It is easily understood, by a consideration of the nature of Equirational Series, that the product of any two extreme terms is equal to the product of any two

intermediate terms equidistant from those extremes, or to the square of the mean term. Thus, in the former of the series exemplified above, $3 \times 48 = 6 \times 24 = 12 \times 12$.

It is also readily observed, that the common ratio involved to a power whose exponent is 1 less than the number of terms, produces the quotient of the greatest term divided by the least. So that if l represent the least term, g the greatest, r the common ratio, and n the number of terms, we have $g \div l = r^{n-1}$, or $g = l \times r^{n-1}$.

Since l, lr, lr^2, lr^3, &c., is the general form of the series now under consideration, and since $lr + lr^2 + lr^3$, or the sum of the series diminished by the least term, is evidently equal to $(l + lr + lr^2) \times r$, that is, the sum of the series diminished by the greatest term, and the remainder multiplied by the common ratio, we have

$$s - l = (s - g) \times r, \text{ or } s \times (r - 1) = gr - l;$$

but g being equal to $l \times r^{n-1}$, we have

$$s \times (r-1) = lr^n - l = l \times (r^n - 1).$$

Hence, we have these two equations:

$$g = l \times r^{n-1}, \text{ and } s = \frac{r^n - 1}{r - 1} \times l,$$

from which to deduce other formulae for the evaluation of any of the elements. But it must be observed, that the value of the exponential element n cannot be conveniently exhibited without the employment of Logarithms.

NOTE.—In an *infinite* descending or decreasing series of proportionals, l is, of course, less than any assignable number, and consequently $= 0$; also n, being infinite, is greater than any assignable number. Hence, in such a case, $s = (gr - l) \div (r - 1)$ becomes $s = gr \div (r - 1)$.

1. Given l, n, r, to find g and s.
$g = l \times r^{n-1}$; $s = (r^n - 1) \div (r - 1) \times l$.

2. Given g, r, n, to find l and s.
$l = g \div r^{n-1}$; $s = (r^n - 1) \div (r - 1) \times g \div r^{n-1}$.

3. Given g, l, n, to find r and s.

$$r = (g \div l)^{\frac{1}{n-1}}; \quad s = g + (g - l) \div [(g \div l)^{\frac{1}{n-1}} - 1].$$

4. Given g, l, r, to find s and n.

$$s = (gr - l) \div (r - 1); \quad n = 1 + (\text{Log. } g - \text{Log. } l) \div \text{Log. } r.$$

5. Given r, n, s, to find l and g.

$$l = (r - 1) \div (r^n - 1) \times s; \quad g = (r - 1) \div (r^n - 1) \times s \times r^{n-1}.$$

6. Given g, l, s, to find r and n.

$$r = (s - l) \div (s - g); \quad n = 1 + (\text{Log. } g - \text{Log. } l) \div [\text{Log. } (s - l) - \text{Log. } (s - g)].$$

7. Given g, r, s, to find l and n.

$$l = gr - (r - 1) \times s; \quad n = 1 + (\text{Log. } g - \text{Log. } [gr - (r - 1) \times s]) \div \text{Log. } r.$$

8. Given l, r, s, to find g and n.

$$g = s - (s - l) \div r; \quad n = (\text{Log. } [l + (r - 1) \times s] - \text{Log. } l) \div \text{Log. } r.$$

9. Given l, n, s, to find g and r.

$$g \times (s - g)^{n-1} = l \times (s - l)^{n-1}; \quad s \div l \times r - r^n = (s - l) \div l.$$

10. Given g, n, s, to find l and r.

$$l \times (s - l)^{n-1} = g \times (s - g)^{n-1}; \quad r^n = (s \times r^{n-1} - g) \div (s - g).$$

Examp. Given the least term of a series of continual proportionals = 3, the common ratio = 4, and the number of terms = 10; to find the greatest term.

By 1st Theor. $g = 3 \times 4^9 = 786432$. *Ans.*

Examp. Required the sum of a decreasing equirational series, of which the last term is $\frac{3}{4^7}$, the ratio 3, and the number of terms 16.

By 1st Theor. $s = (3^{16} - 1) \div 2 \times \frac{3}{4^7}$;

$= 43046720 \div 27 = 1594322\frac{2}{3}\frac{6}{7}$. *Ans.*

Examp. What is the sum of an infinitely continued equirational series, of which the first or greatest term is 2, and each succeeding term $\frac{1}{3}$ of the preceding?

Here, the greatest term is 2, the least term 0, and the ratio $\frac{1}{3}$ or $2\frac{1}{3}$; wherefore by 4th Theor. $s = (2 \times 2\frac{1}{3} - 0) \div 1\frac{1}{3} = 3\frac{1}{2}$. *Ans.*

Examp. Required the value of the interminate decimal $\cdot 1\dot{4}\dot{5}$.

The circulating part of the decimal consists of 45

thousandths + 45 *hundred-thousandths* + 45 *ten-millionths*, &c., to infinity. This forms an infinite series having $g = \cdot045$, $l = 0$, $r = 100$. Accordingly, by 4th Theor.

$s = \cdot045 \times 100 \div 99 = \frac{1}{22}$; therefore $\cdot\dot{1}\dot{4}\dot{5} = \frac{1}{10} + \frac{1}{22} = \frac{8}{55}$. *Ans.*

Examp. Find a mean proportional between $2777\frac{1}{7}$ and $99977\frac{1}{7}$.

This is to find the second of three terms of an equirational series. Now, by 3rd Theor. the common ratio is $(g \div l)^{\frac{1}{2}} = \sqrt{36} = 6$; therefore $2777\frac{1}{7} \times 6 = 16662\frac{4}{7}$. *Ans.*

Examp. The sum of an equirational series is 115·73415, the least term being 15 and the common ratio $1\frac{1}{10}$. Find the number of terms.

According to 8th Theor. $l + (r - 1) \times s = 26\cdot573415$; $n = (\text{Log. } 26\cdot573415 - \text{Log. } 15) \div \text{Log. } 1\cdot1$; $(1\cdot4244474 - 1\cdot1760913) \div \cdot0413927 = 6$. *Ans.*

Examp. A servant draws off one gallon each day, for 5 days, from a cask containing 13 gallons of brandy, each time supplying the deficiency by a gallon of water; and then, to escape detection, he draws off 5 gallons as before, supplying the deficiency each time by a gallon of brandy. Required how much water remains in the cask.

In the first instance, the brandy decreases continually as 13 : 12. Accordingly, we have $g = 13$, $n = 6$, $r = \frac{12}{13}$; to find l, the quantity of brandy in the cask at the end of 5 days.

By 2nd Theor. $l = 13 \times (\frac{12}{13})^5 = 13 \times \cdot670177$; or 8·7123 gall. B. + 4·2877 gall. W. left at the end of the first 5 days.

In the second instance, the water decreases continually as 13 : 12; whence, by similar determination, $4\cdot2877 \times \cdot670177 = 2\cdot8735$ gall. water. *Ans.*

When an equirational series is extended beyond an integral number of terms, to include a fractional approximation to the next term, the assigned fraction denotes the multiplication of a radical part of the common ratio; as in the annexed example, which finds the sum of $4\frac{1}{2}$ terms of the series 7, 35, &c.

1st term	=	7
2nd,	7 × 5 =	35
3rd,	35 × 5 =	175
4th,	175 × 5 =	875
$4\frac{1}{2}$th,	875 × $5^{\frac{1}{2}}$ =	1956·5595
	Sum,	3048·5595

When the last term of an equirational series is thus incomplete, several of the Theorems are inapplicable.

Examp. Required the sum of $4\frac{1}{2}$ terms of that equirational ascending series whose least term is 7, and common ratio 5.

By 1st Theor. the $4\frac{1}{2}$th term $= 7 \times 5^{4\frac{1}{2}} = 7 \times 5^4 \times \sqrt{5}$;

Sum of 4 terms $= (5^4 - 1) \div 4 \times 7 = 1092$

$g = 1956{\cdot}5595$

Ans. $s = 3048{\cdot}5595$

Examp. In a broken ascending series, given $l = 7$, $g = 1956{\cdot}5595$, and $r = 5$; to find n.

By 4th Theor. 1 + (Log. g, 3·291493 − Log. l, ·845098) ÷ Log. $r = 1 + 2{\cdot}446395 \div {\cdot}69897 = 4\frac{1}{2}$. *Ans.*

EXERCISES.

1. If £136 be the least term of an equirational series, 1·05 the common ratio, and 12 the number of terms; what is the greatest term? *Ans.* £232. 12*s.* 1½*d.*

2. Given, the greatest term 708588, the ratio 3, and the number of terms 12; to find the least term. *Ans.* 4.

3. Given, the extremes 3 and 88, and the common ratio 5; to find the number of terms. *Ans.* 3·09932.

4. Given, the number of terms 3, and the extremes 4 and 5; to find the second term or mean proportional. *Ans.* 4·47214.

5. The 3rd term in a series of continual proportionals is $230\frac{7}{8}$, and the 12th term is 118208. Required the 6th term. *Ans.* 1847.

6. Find the sum of the infinite series 4, 1, $\frac{1}{4}$, &c.
Ans. $5\frac{1}{3}$.

7. Find the values of the decimals $\cdot 0\dot{2}\dot{7}$ and $\cdot 4\dot{5}1\dot{8}$.
Ans. $\frac{1}{3\,7}$; $\frac{6\,1}{1\,3\,5}$.

8. Required the sum of $\frac{1}{10} + \frac{2}{10^2} + \frac{1}{10^3} + \frac{2}{10^4}$, &c., to infinity. *Ans.* $\frac{4}{33}$.

9. Find two mean proportionals between 5·45 and 1869·35. *Ans.* 38·15 and 267·05.

10. Given, the greatest term 2·1, the ratio 1·004074, and the sum of the series 26·646. Required the least term, and the number of terms. *Ans.* 2; 13.

11. The population of a parish in 1821 was 2180, and in 1831 it was 2376. Required the population in 1841; supposing the rate of increase at any time to be proportional to the population at that time. *Ans.* 2590.

12. A country contains one million of inhabitants. How many did it contain a century ago, supposing the population to have been doubled every 20 years?
Ans. 31250.

13. If a body be put in motion by a force which moves it 10 miles in the first portion of time, 9 miles in the second equal portion, and so on in the ratio of 10 : 9 for ever; how many miles will it pass over? *Ans.* 100.

14. A and B set out from London at the same time, and travelled in the same direction. On each succeeding day, A travelled $1\frac{1}{10}$ as many miles and B $\frac{4}{5}$ as many as they did the preceding day; and when B had finished his 7th day's journey, which was exactly the same as A's 1st day's journey, he was 24 miles in advance of A. How far did each travel the first day?
Ans. A 7, B 20·902 mi.

15. In a broken series of continual proportionals, the least term is $4\frac{1}{2}$, the common ratio $5\frac{1}{2}$, and the number of terms $6\frac{1}{2}$. Required the greatest or broken term.
Ans. 37756·345.

16. The sum of an incomplete ascending series is

863624840, the number of terms $5\frac{1}{4}$, and the ratio 81; find the least term. *Ans.* 5.

17. Suppose a certain number of persons to increase every year by a tenth part; in what time will there be ten times as many persons as there are now?

Ans. 24·1588 yrs.

18. A merchant gains in one year £729, in the next year $\frac{2}{3}$ of that sum, in the third he gains $\frac{2}{3}$ of what he gains the second, and so on uniformly. Find the total amount of his gains at the end of 4 years 3 months.

Ans. £2962. 2*s.* 10*d.*

Compound Interest.

Compound Interest is interest calculated on the Amount of Principal and Simple Interest. Thus, if I lend £450 at 5 per cent. per annum, and if at the end of the first year, having obtained £22. 10*s.* of interest, I lend £450 + £22. 10*s.* for the next year, I shall obtain £23. 12*s.* 6*d.* as the second year's interest; if the amount £496. 2*s.* 6*d.* is made Principal for the year following, the interest will be £24. 16*s.* $1\frac{1}{2}d.$ By this means the amount of £450 is £472·5 at the end of the 1st year, £496·125 at the end of the 2nd, £520·93125 at the end of the 3rd.

Now, the successive amounts by compound interest form a series of continual proportionals, the first term being the original principal, the number of terms, *minus* 1, being the number of periods at which the interest is payable, and the common ratio being 1 increased by the fraction which proportionally expresses the interest. This will readily appear from the following example.

Examp. What will be the amount of £750, at the end of 3 years, at 4 per cent. per ann. comp. int.?

Solution in two forms.

25)750		750
30		1·04
25)780	amount at end of 1st year	780
31·2		1·04
25)811·2	amount at end of 2nd year	811·2
32·448		1·04
£843·648	*Ans.*	£843·648

In these forms of solution, we see each amount to be $1\frac{1}{25}$, or 1·04, of the amount preceding; but the second form suggests a mode of solution more expedient than either, as it exhibits a series of continual proportionals, in which the first term is 750, the number of terms (*minus* 1) is 3, and the common ratio, 104 ÷ 100, or 1·04. Accordingly, $1{\cdot}04^3 \times 750 = 1{\cdot}124864 \times 750$ = £843·648, the 4th term, or the amount at the end of 3 years.

Observe, that £1·04 is the amount of £1 for 1 year, and that when it is increased by $\frac{1}{25}$ of itself, or multiplied by 1·04, the product must be the amount of £1 for 2 years, and that amount × 1·04 gives the amount for 3 years, &c. Accordingly, the amount of £1 at any rate per cent. compound interest is equal to (1 + the rate of interest per £) involved to the power denoted by the number of terms of payment; and the amount for any number of £s is the product of that number by the amount of £1.

The sum of interest is found by deducting the original principal from the final amount; or by multiplying the sum of the interest of £1 by the original principal.

Examp. What is the comp. int. on £1234. 15*s.* 6$\frac{3}{4}$*d.* for 10 years, at 4$\frac{1}{2}$ p. c. per ann.?

Solution in two forms.

$1{\cdot}045^{10}$ =	1·5529694	Or, Int. =	·5529694
	1234		1234
	1916·36424		682·36424
$10s. = \frac{1}{2}$	·77648		·27648
$5s. = \frac{1}{4}$	·38824		·13824
$6d. = \frac{1}{10}$	·03882		·01382
$\frac{1}{4}d. = \frac{1}{96}$	·00486		·00173
Amt.	£1917·57264	Int.	£682·79451

£1917. 11*s.* 5½*d.* − £1234. 15*s.* 6¾*d.* = £682. 15*s.* 10¾*d.* *Ans.*

EXERCISES.

Find the amount, at comp. int., of

	£.	*s.*	*d.*	Yrs.	P.C.	*Answers.*
1.	238	10	0	for 5	at 2	£263 6*s.* 5·585*d.*
2.	560	18	0	6	4	709 14 4·185
3.	394	6	6	8	2½	480 8 11·214
4.	1000	0	0	10	5	1628 17 10·710
5.	875	12	2½	9	4½	1273 9 9·567

Find the compound interest on

6.	300	0	0	7	3½	81 13 8·107
7.	156	13	4	16	5½	212 6 5·877
8.	0	6	6	20	8	1 3 9·555

Compound Interest for Parts of a Year.

Compound Interest being payable yearly, at an assigned rate per cent., it may be required to convert that rate into an equivalent rate per cent. per half-year, per quarter, &c.

Now, although simple interest, at a specified rate per cent. per annum, may be calculated at half that rate per cent. per half-year, it is otherwise with compound interest. Amounts by simple interest arise in *arithmetical*

progression; those by compound interest arise in *geometrical* progression. The amount of £100 in 2 years at 4 per cent. per ann. simple interest results from a yearly *addition* of £4 or from a half-yearly addition of £2, [*i. e.* the arithmetical mean between £0, interest at the outset, and £4 interest at the end of a year, $= (0 + 4) \div 2$]. The amount of £100 in 2 years at 4 per cent. per ann. compound interest results from a yearly *product* by 1·04, or from a half-yearly product by 1·0198 [*i. e.* the geometrical mean between £1, amount at the outset, and £1·04, amount at the end of a year, $= (1 \times 1{\cdot}04)^{\frac{1}{2}}$]. The comparison may be exhibited as follows:—

	Simple.		*Compound.*
Amt. at end of 1st year	$100 + 4$	$=$	$100 \times 1{\cdot}04$
Ditto 2nd	$104 + 4$		$104 \times 1{\cdot}04$
By yearly payments	108		108·16

	Simple.		*Compound.*
Amt. at end of 1st hf.-yr.	$100 + \frac{1}{2}$ of 4	100	$\times 1{\cdot}04^{\frac{1}{2}}$
Ditto 2nd	$102 + \frac{1}{2}$ of 4	$=$ 101·98	$\times 1{\cdot}04^{\frac{1}{2}}$
Ditto 3rd	$104 + \frac{1}{2}$ of 4	104	$\times 1{\cdot}04^{\frac{1}{2}}$
Ditto 4th	$106 + \frac{1}{2}$ of 4	106·0592	$\times 1{\cdot}04^{\frac{1}{2}}$
By half-yearly payments	108		108·16

It appears, then, that simple interest is greater than compound interest for the fractional parts of a year; and this may, on first view, be thought unreasonable. But it should be observed that, when interest of money is drawn at an earlier period than that at which it is due, the sum received should be smaller in proportion as its gain by the kind of investment to which it belongs would be greater.

In the calculation of compound interest, therefore, we must uniformly raise the given amount of £1 for one term of payment to the power whose exponent is the number of terms. As the amount of £1 at 4 p. c. per ann. for 2 years is $1{\cdot}04^2$, so the amount for $\frac{1}{2}$ of a year is $1{\cdot}04^{\frac{1}{2}}$; for $\frac{1}{4}$ of a year, $1{\cdot}04^{\frac{1}{4}}$; for 1 day, $1{\cdot}04^{\frac{1}{365}}$; for 53 days, $1{\cdot}04^{\frac{53}{365}}$; for $5\frac{1}{4}$ yrs. it is $1{\cdot}04^5 \times 1{\cdot}04^{\frac{1}{4}}$.

In many works on Arithmetic such questions as the following

occur:—"Required the amount of £150 by compound interest in 3 years at 5 p. c. per ann., *interest payable quarterly.*" Here we must understand that interest is required for 12 quarters, at the rate of $1\frac{1}{4}$ p. c. per quarter, payable quarterly. It is true that 1·05¼, or 1·01227 would be the quarterly amount of £1, on the supposition of the interest, 5 p. c. per ann., being *payable yearly;* but that is not the datum in the question; and, therefore, it seems unnecessary to condemn, as Kavanagh has done,* the usual mode of enunciating such problems, since 5 p. c. is not meant to denote the rate to be applied in calculation, but is merely a number suggesting that rate.

EXERCISES.

9. Find the amount of £256, for $\frac{1}{4}$ a year, at 3 p. c. per ann. comp. interest, payable yearly.

Ans. £259. 16*s.* $2\frac{3}{4}$*d.*

10. Find the comp. interest on £45. 15*s.* for $\frac{1}{4}$ of a year, at $3\frac{1}{2}$ p. c. per ann., payable yearly.

Ans. £1. 2*s.* $2\frac{1}{2}$*d.*

11. Required the amount of £300, for $\frac{1}{2}$ of a year, at $4\frac{1}{2}$ p. c. per ann. comp. interest, payable yearly.

Ans. £304. 8*s.* $8\frac{1}{2}$*d.*

12. Compute the amount of £1650, for $2\frac{1}{4}$ yrs. at 5 p. c. per ann. comp. interest, payable half-yearly.

Ans. £1866. 16*s.* $5\frac{1}{4}$*d.*

13. Calculate the comp. interest on 54 guineas, for 3 years at 5 p. c. per ann., payable quarterly.

Ans. £9. 2*s.* $3\frac{1}{4}$*d.*

14. What is the amount of £98. 12*s.* 3*d.*, for $3\frac{1}{4}$ years, at 4 p. c. per ann. comp. interest?

Ans. £112. 0*s.* $4\frac{1}{2}$*d.*

15. What is the comp. interest of £217 for $2\frac{1}{2}$ years at 5 p. c. per ann., interest payable half-yearly?

Ans. £25. 10*s.* $0\frac{3}{4}$*d.*

Note:—The above examples and exercises contain, as data, the principal, rate, and time, to find the amount or the interest. We may be required, however, to find what principal, for a given rate and time, will produce a given sum of interest; also, in what time a given principal, at a given rate, will produce a given amount; &c. The solution of such questions is obviously suggested by the direct rule already explained.

* *Arithmetic*, p. 120. Dublin, 1844.

Examp. What principal, improved for 3 years at 4 p. c. per ann. comp. interest, will amount to £100?

The princ. $\times 1{\cdot}04^3$ is to be £100; therefore, £100 $\div 1{\cdot}04^3$ must give the principal; or, £100 $\div$ 1·124864 = £88. 18*s.* *Ans.*

Examp. At what rate p. c. per ann. comp. interest will £400 amount to £500 in 10 years?

$500 \div 400 = 1{\cdot}25$, the 10th power of the ratio. $\sqrt[10]{1{\cdot}25} = 1{\cdot}022565$, or 2·2565 p. c. *Ans.*

Examp. In how many years will £560 become £650, by comp. interest at 5 p. c. per ann., payable quarterly?

$650 \div 560 = 1{\cdot}1607143$, the required amount of 1·0125 involved to the power denoted by the number of quarters of a year, which number may be found by making trial of 11½ qrs. and 12 qrs.; or else logarithms may be employed. The result will be found = 11·9972 qrs., or very nearly 3 yrs. *Ans.*

Examp. A property of £10000 is left to 4 children, whose ages are 6, 8, 10, and 12 years, respectively; and it is so divided among them, that their several shares being improved at 4½ p. c. per ann. comp. int., they shall all have equal properties at the age of 21. What is the sum left to each?

As the common difference of the ages is 2, the shares are as 1, $1{\cdot}045^2$, $1{\cdot}045^4$, and $1{\cdot}045^6$; that is, as 1, 1·092025, 1·1925186, and 1·302260125; and £10000 being divided according to that proportion, gives the several shares £2180. 3*s.* 4¼*d.*, £2380. 15*s.* 11¼*d.*, £2599. 17*s.* 9¼*d.*, and £2839. 2*s.* 10¾*d.* *Ans.*

EXERCISES.

16. Required the present worth of £325. 7*s.* 6*d.* for 8 years at 5 p. c. per ann. comp. int. *Ans.* £220. 4*s.* 6½*d.*

17. What principal will amount to £365 in 12 years at 4 p. c. per ann. comp. int., payable half-yearly?

Ans. £226. 18*s.* 6¾*d.*

18. In what time will £1000 amount to £2500 at 3 p. c. per ann. comp. int.? *Ans.* 31 yrs. *nearly.*

19. Find the present worth of £4½ for 4½ years, at 4½ p. c. per ann. *Ans.* £3. 13*s.* 10*d.*

20. In what time would any sum of money gain one-sixth of itself, by comp. int. at 6 p. c. per ann.? *Ans.* 2·6455 yrs.

21. At what rate p. c. per ann. comp. int. would £240 amount to £275. 11*s.* 10*d.* in 4 years? *Ans.* 3·5175 p. c.

22. At what rate p. c. per ann. comp. int. would 1 penny become £1 in 111 years, interest payable every 2 years? *Ans.* 5·1895 p. c.

23. If I commence trade with a certain capital, and find that, by applying the profit at the end of each year to the same trade at the beginning of the next, my stock in 7 years has been doubled. Required my gain p. c. per ann. *Ans.* 10·409 p. c.

24. In what time will the amount of £254 at 4 p. c. per ann. equal the amount of £286 at 3 p. c. per ann. comp. int.? *Ans.* 12·281 yrs.

25. What sum will amount to 25*s.* 6*d.* more at simple than at compound interest, in 4 months, at 3 p. c. per ann., payable yearly? *Ans.* £12961. 16*s.*

26. Divide £1000 into three sums, such that their compound interests, at 5, 4, and 3 p. c. per ann. respectively, for 6, 7, and 8 years, may be equal sums. *Ans.* £298. 7*s.* 9*d.*; £321. 4*s.* 2¼*d.*; £380. 8*s.* 0¼*d.*

27. What sum of money, improved by comp. int., amounts to £2. 10*s.* 9¼*d.* more in 4½ years at 4 p. c. than in 4 years at 4½ p. c.? *Ans.* £5000.

28. At what rate p. c. per ann. would the compound amount of £12 for 10 years be 19 *twentieths* of the compound amount of £10 for 12 years? *Ans.* 12·3903 p. c.

Note:—The involution of the ratios 1·03, 1·04, &c., being often a very laborious process, we have furnished Tables of their amounts, which will greatly facilitate the solution of the more useful kind of Compound Interest questions. To promote acquaintance with the use of these Tables, some of the preceding exercises may now be wrought by their assistance. Tables of Logarithms are also a very serviceable means for the solution of problems in Compound Interest.

Amount of £1 *for Years.*

Yrs.	2½ *per cent.*	3 *per cent.*	3½ *per cent.*
1	1·025	1·03	1·035
2	1·050625	1·0609	1·071225
3	1·07689062	1·092727	1·10871787
4	1·10381289	1·12550881	1·14752300
5	1·13140821	1·15927407	1·18768631
6	1·15969342	1·19405230	1·22925533
7	1·18868575	1·22987387	1·27227926
8	1·21840290	1·26677008	1·31680904
9	1·24886297	1·30477318	1·36289735
10	1·28008454	1·34391638	1·41059876
11	1·31208666	1·38423387	1·45996972
12	1·34488882	1·42576089	1·51106866
13	1·37851104	1·46853371	1·56395606
14	1·41297382	1·51258972	1·61869452
15	1·44829817	1·55796742	1·67534883
16	1·48450562	1·60470644	1·73398604
17	1·52161826	1·65284763	1·79467555
18	1·55965872	1·70243306	1·85748920
19	1·59865019	1.75350605	1·92250132
20	1·63861644	1·80611123	1·98978886
21	1·67958185	1·86029457	2·05943147
22	1·72157140	1·91610341	2·13151158
23	1·76461068	1·97358651	2·20611448
24	1·80872595	2·03279411	2·28332849
25	1·85394410	2·09377793	2·36324498
26	1·90029270	2·15659127	2·44595856
27	1·94780002	2·22128901	2·53156711
28	1·99649502	2·28792768	2·62017196
29	2·04640739	2·35656551	2·71187798
30	2·09756758	2·42726247	2·80679370
$\frac{1}{12}$	1·00205984	1·00246627	1·00287090
$\frac{1}{4}$	1·00619225	1·00741707	1·00863745

Amount of £1 *for Years*—continued.

Yrs.	2½ *per cent.*	3 *per cent.*	3½ *per cent.*
31	2·15000677	2·50008035	2·90503148
32	2·20375694	2·57508276	3·00670759
33	2·25885086	2·65233524	3·11194235
34	2·31532213	2·73190530	3·22086033
35	2·37320519	2·81386245	3·33359045
36	2·43253532	2·89827833	3·45026611
37	2·49334870	2·98522668	3·57102543
38	2·55568242	3·07478348	3·69601132
39	2·61957448	3·16702698	3·82537171
40	2·68506384	3·26203779	3·95925972
41	2·75219043	3·35989893	4·09783381
42	2·82099520	3·46069589	4·24125799
43	2·89152007	3·56451677	4·38970202
44	2·96380808	3·67145227	4·54334160
45	3·03790328	3·78159584	4·70235855
46	3·11385086	3·89504372	4·86694110
47	3·19169713	4·01189503	5·03728404
48	3·27148956	4·13225188	5·21358898
49	3·35327680	4·25621944	5·39606459
50	3·43710872	4·38390602	5·58492686
51	3·52303644	4·51542320	5·78039930
52	3·61111235	4·65088590	5·98271327
53	3·70139016	4·79041247	6·19210824
54	3·79392491	4·93412485	6·40883202
55	3·88877303	5·08214859	6·63314114
56	3·98599236	5·23461305	6·86530108
57	4·08564217	5·39165144	7·10558662
58	4·18778322	5·55340098	7·35428215
59	4·29247780	5·72000301	7·61168203
60	4·39978975	5·89160310	7·87809090
¼	1·01242284	1·01488916	1·01734950
¾	1·01869201	1·02241666	1·02613680

Amount of £1 *for Years*—continued.

Yrs.	4 *per cent.*	4½ *per cent.*	5 *per cent.*
1	1·04	1·045	1·05
2	1·0816	1·092025	1·1025
3	1·124864	1·14116612	1·157625
4	1·16985856	1·19251860	1·21550625
5	1·21665290	1·24618194	1·27628156
6	1·26531902	1·30226012	1·34009564
7	1·31593178	1·36086183	1·40710042
8	1·36856905	1·42210061	1·47745544
9	1·42331181	1·48609514	1·55132822
10	1·48024428	1·55296942	1·62889463
11	1·53945406	1·62285305	1·71033936
12	1·60103222	1·69588143	1·79585633
13	1·66507351	1·77219610	1·88564914
14	1·73167645	1·85194492	1·97993160
15	1·80094351	1·93528244	2·07892818
16	1·87298125	2·02237015	2·18287459
17	1·94790050	2·11337681	2·29201832
18	2·02581652	2·20847877	2·40661923
19	2·10684918	2·30786031	2·52695020
20	2·19112314	2·41171402	2·65329771
21	2·27876807	2·52024116	2·78596259
22	2·36991879	2·63365201	2·92526072
23	2·46471554	2·75216635	3·07152376
24	2·56330416	2·87601383	3·22509994
25	2·66583633	3·00543446	3·38635494
26	2·77246978	3·14067901	3·55567269
27	2·88336858	3·28200956	3·73345632
28	2·99870332	3·42969999	3·92012914
29	3·11865145	3·58403649	4·11613560
30	3·24339751	3·74531813	4·32194238
$\frac{1}{52}$	1·00327374	1·00367481	1·00407412
½	1·00985341	1·01106499	1·01227223

Amount of £1 *for Years*—continued.

Yrs.	4 *per cent.*	4½ *per cent.*	5 *per cent.*
31	3·37313341	3·91385745	4·53803949
32	3·50805875	4·08998104	4·76494147
33	3·64838110	4·27403018	5·00318854
34	3·79431634	4·46636154	5·25334797
35	3·94608899	4·66734781	5·51601537
36	4·10393255	4·87737846	5·79181614
37	4·26808986	5·09686049	6·08140694
38	4·43881345	5·32621921	6·38547729
39	4·61636599	5·56589908	6·70475115
40	4·80102063	5·81636454	7·03998871
41	4·99306145	6·07810094	7·39198815
42	5·19278391	6·35161548	7·76158756
43	5·40049527	6·63743818	8·14966693
44	5·61651508	6·93612290	8·55715028
45	5·84117568	7·24824843	8·98500779
46	6·07482271	7·57441961	9·43425818
47	6·31781562	7·91526849	9·90597110
48	6·57052824	8·27145557	10·40126965
49	6·83334937	8·64367107	10·92133313
50	7·10668335	9·03263627	11·46739979
51	7·39095068	9·43910490	12·04076978
52	7·68658871	9·86386463	12·64280826
53	7·99405226	10·30773853	13·27494868
54	8·31381435	10·77158677	13·93869611
55	8·64636692	11·25630817	14·63563092
56	8·99222160	11·76284204	15·36741246
57	9·35191046	12·29216993	16·13578309
58	9·72598688	12·84531758	16·94257224
59	10·11502635	13·42335687	17·78970085
60	10·51962741	14·02740793	18·67918589
¼	1·01980390	1·02225242	1·02469508
¾	1·02985245	1·03356363	1·03727037

ANNUITIES.

ANNUITIES are fixed annual incomes or payments, and are of two principal kinds, *Certain* and *Contingent*.

An *Annuity Certain* commences at a definite period; an *Annuity Contingent* is one whose term depends on some contingency, as the continuance of a life.*

Amount of an Annuity.

The *Amount* of an Annuity is the sum to which it will amount, if each payment is laid up, as it becomes due, and improved by compound interest.

Examp. Required the amount of an annuity of £70, unpaid, or in arrears, for 6 years, at 5 p. c. per ann. comp. int.

The 6th year's payment is	£70
5th ditto with 1 year's int.	73·5
4th ditto with 2 ditto.	77·175
3rd ditto with 3 ditto.	81·03375
2nd ditto with 4 ditto.	85·085438
1st ditto with 5 ditto.	89·339709
Amt. due at the end of 6 yrs.	£476·133897.

Now, this result might have been more conveniently obtained by Theorem 1st of Equirational Series; £70 being the first or least term of the series, 1·05 the common ratio, and 6 the number of terms; to find the sum of the series.

$$s = [(r^n - 1) \div (r - 1)] \times l.\dagger$$

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* An excellent compendium of the subject of Life Annuities will be found in Nesbit's 2nd Vol. of *Arithmetic*.

† In all the Examples and Exercises we have made the number of terms of payment *integral*. Some books on Arithmetic contain Problems that require the duration of an annuity, when the period is not purely integral; the answers given are erroneous, because adapted to the above formula, which is applicable only to integral values of n.

Accordingly, the computation may proceed as follows:—

Amt. of £1 for 6 yrs. $= 1{\cdot}05^6$,	£1·34009564
	1
Int. of £1 for 1 and 6 yrs.,	·05) ·34009564
Amt. of £1 per ann. in 6 yrs.,	6·8019128
	70
Amt. of £70 per ann. in 6 yrs.,	£476·133897

Or, £476. 2*s*. 8·135*d*. *Ans*.

Note:—If the first payment were to be received *immediately*, the amount at the end of 6 years would evidently be equal to the above answer × 1·05. But it is always understood that the first payment is made at the end of the first of those terms for which the valuation is made.

Examp. Required the amount of an annuity of £70, payable quarterly, in 6 years, int. 5 p. c. per ann.

In problems of this description, it is generally assumed in practice, that the interest as well as the annuity is payable quarterly, that is, that the rate of interest is 1¼ per cent. per quarter. On this supposition, the solution will be equivalent to that which determines the amount of an annuity of £17·5, for 24 years, at 1¼ p. c. per ann.

$$\frac{1{\cdot}0125^{24} - 1}{{\cdot}0125} \times 17{\cdot}5 = \frac{{\cdot}34735105}{{\cdot}0125} \times 17{\cdot}5,$$

$= £486{\cdot}29147$, or £486. 5*s*. 10*d*. *Ans*.

The amount in this case turns out £10. 3*s*. 2*d*. more than if the annuity and interest had been payable yearly; and it is evident that the more frequent the payments are, the greater will be the amount, for the several gains by interest are thus put sooner to gain more interest.

But if it be understood that the money arising from the annuity is improved at an interest of 5 per cent. *payable yearly*, then, according to the true rate of annual interest, the amount of £1 for a quarter of a year,

at 5 p. c. per ann., will be (not 1·0125, but) $1·05^{\frac{1}{4}} = 1·01227223$; and the amount of the given annuity will be

$$\frac{1·05^{6} - 1}{·01227223} \times 17·5 = £484·97067.$$

Examp. Required the amount of an annuity of £70, for 6 years, improved at the rate of 5 p. c. per ann. compound interest; the annuity being payable half-yearly, and the interest quarterly.

In solving this problem, it is better to consider it as requiring the amount of an annuity of £35, payable yearly, for 12 years, at $2\frac{1}{2}$ p. c. per ann. interest payable half-yearly.

Then the 12th year's payment is £35; the 11th ditto, with interest for 2 half-yearly terms, is £35 × $1·0125^{2}$; the 10th ditto, with interest for 4 half-yearly terms, is £35 × $1·0125^{4}$; &c.

The data of the progression, therefore, are $l = 35$, $r = 1·0125^{2}$, $n = 12$; to find s.

Accordingly, $s = (1·0125^{24} - 1) \div (1·0125^{2} - 1) \times 35$; or, $(·34735105 \div ·02515625) \times 35 = £483·271026$. *Ans.*

We may here notice another form of solution, which is sometimes employed in finding the amount of an annuity.*

Let it be required again, to find the amount of an annuity of £70, for 6 years, at 5 p. c. per ann.

The principal which would gain £70 per ann., at 5 p. c. *simple interest*, is £1400; and its *compound interest*, at 5 p. c. for 6 years, is $·340095641 \times 1400 = £476·133897$, which is likewise the amount of the annuity.

Note:—Problems such as the one following involve an application of the principles employed in the computation of annuities.

Examp. Suppose the acting partner in a commercial company to contribute £500 to the original capital of the firm, and to increase this sum annually by £90 saved from his salary; what

* See Malcolm's *Arithmetic*, p. 606; or Thomson's *Arithmetic*, 23rd ed., p. 215.

would be his share of the joint stock at the end of 8 years, reckoning the clear gain on the entire capital to be $16\frac{2}{3}$ p. c. per ann.?

$$\pounds 500 \times (1\tfrac{1}{6})^8 = \pounds 1716 \cdot 10684$$
$$\pounds 90 \times [(1\tfrac{1}{6})^8 - 1] \div \tfrac{1}{6} = 1313 \cdot 39538$$

Ans. £3029·50222

In the above solution are calculated, first, the amount of the entry-stock, £500, for 8 years, at $16\frac{2}{3}$ p. c.; then, that of the annual contribution, £90, for the same time, and at the same rate; and the sum of both amounts is the share of stock required.

EXERCISES.

1. Required the amount of an annuity of £100, unpaid for 11 years, at 4 per cent. per ann.
Ans. £1348. 12*s.* 8·43*d.*

2. Calculate the arrear of an annuity of £436, for 18 years, at 6 p. c. *Ans.* £13474. 17*s.* 3·48*d.*

3. Find the amount of an annuity of £56. 10*s.* 6*d.*, forborn 13 years, at $4\frac{1}{2}$ p. c. *Ans.* £969. 19*s.* 3·38*d.*

4. Find the amount of an annuity of £100, forborn 51 years, at 5 p. c. *Ans.* £22081. 10*s.* 9·49*d.*

5. Compute the arrear of a yearly salary of £104, for 11 years at 5 p. c. per ann., both salary and interest being payable semi-annually. *Ans.* £1500. 17*s.* 4·44*d.*

6. If my annual salary of £60 lie over in arrears for 4 years; to what will it then amount, at 4 p. c., both salary and interest being payable quarterly?
Ans. £258. 17*s.* 4·31*d.*

7. Required the amount of £10 per ann., 24 years in arrear, but payable only every second year, interest 5 p. c. per ann., payable also every second year.
Ans. £427. 13*s.* 8·56*d.*

8. What is the amount of an annuity of £60, for 5 years, at 5 p. c., the annuity being payable half-yearly, and the interest quarterly? *Ans.* £336. 6*s.* 10·21*d.*

9. If my annual salary of £100, which is payable quarterly, be unpaid for 4 years; to what will it amount, reckoning interest at 5 p. c., payable yearly?
Ans. £439. 0*s.* 2·84*d.*

10. If a yearly pension of £30, payable monthly, be forborn 8 years; what sum will be due at the end of that period, the interest, 4½ p. c., being convertible into principal yearly? *Ans.* £287. 3*s.* 1·96*d.*

11. A merchant puts out to compound interest the sum of £350, and increases the sum annually by £50. Required the total amount at the end of 12 years, int. 3½ p. c. per ann. *Ans.* £1258. 19*s.* 5·31*d.*

12. If I enter with a capital of £8000 into a trade which yields a profit of 10 p. c. per ann., and continually apply each year's profit to the same trade, at the beginning of the next; to what will my stock amount at the end of 6 years, supposing my yearly expense to be £150? *Ans.* £13015. 2*s.* 11·16*d.*

13. A person, possessed of a certain capital, lays it out at an advantage of £50 p. c. per ann. compound interest, but withdraws £250 at the end of each year, for the support of his family; by which means he finds, at the end of 9 years, that his capital is 20 times its original amount. Required the original amount.
Ans. £1015. 1*s.* 9·73*d.*

14. What annuity to continue 10 years, will at the end of that period raise a stock of £500, interest 4 p. c.?
Ans. £41. 12*s.* 10·91*d.*

15. What annuity, payable half-yearly, will amount to £425. 6*s.* 9¾*d.*, in 9 years, interest 5 p. c. per ann., payable also half-yearly. *Ans.* £38.

16. What annuity, payable quarterly, will amount to £444, in 44 years, interest 4 p. c. payable yearly?
Ans. £3. 15*s.* 9·76*d.*

17. What annuity, improved at 5 p. c. comp. int., will at the end of 6 years amount to £485. 16*s.* 10·44*d.*, the annuity being payable quarterly, and the interest half-yearly? *Ans.* £70.

Present Value of an Annuity.

The *Present Value* of an Annuity is a sum which, if improved by compound interest for the time of the annuity's continuance, will equal the amount of the annuity for that time.*

Accordingly, as the amount of an annuity is $[(r^n - 1) \div (r - 1)] \times l$, the Present Value must be that amount divided by r^n, viz.

$$[(1 - \frac{1}{r^n}) \div (r - 1)] \times l.$$

Examp. What sum, presently put out at compound interest, will in 4 years overtake the amount of an annuity of £84, to continue during that period, the interest being at 3½ p. c. per ann.?

$1 \cdot 035^4 = 1 \cdot 147523)1 \cdot 00000000$	
Pres. Val. of £1	$\cdot 87144223$
Int. of Pres. Vals. $\cdot 035)$	$\cdot 12855777$
Pres. Val. of £1 per ann.	$3 \cdot 6730792$
	84
Ans.	£308·53865

Examp. What annuity, to continue 4 years, may be purchased for £308. 10*s.* 9·28*d.*, reckoning compound interest at 3½ p. c. per ann.?

The present value of an annuity of £1 is found above to be £3·6730792, by which the given present value being divided, the annuity is found to be £84. *Ans.*

Note 1. To find the present worth of a perpetual annuity, we require only to divide the annuity by the interest of £1 for one term; because, n being infinite, $1 \div r^n$ is $= 0$.

* If the Present Value of an Annuity be divided by the annual payment, the quotient is commonly termed the *number of years' purchase* the Annuity is worth.

Examp. Find the present value of a perpetuity of £70 per ann., reckoning 6 per cent. compound interest.

$$70 \div \cdot 06 = £1166.\ 13s.\ 4d.\ \textit{Ans.}$$

NOTE 2. A reversionary annuity is one which does not come into possession till after a certain interval; and to find its present value, we must first find the value of the annuity at the time when it comes into possession, and then the present value of this sum for the time before it comes into possession.

Examp. What fine must now be paid, to add 25 years to a lease which brings a profit rent of £112. 10*s.*, and of which 14 years are unexpired, compound interest being allowed at 5 p. c. per ann.?

$$1 \cdot 05^{25} = 3 \cdot 38635494)1 \cdot 00000000$$
$$\cdot 29530277$$
$$\cdot 05)\ \cdot 70469723$$
$$14 \cdot 0939446$$
$$112 \cdot 5$$
$$1 \cdot 05^{14} = 1 \cdot 9799316)1585 \cdot 56877$$

Pres. Val. of Reversion, £800·81997 *Ans.*

Examp. Required the present value of a freehold estate of £300 a year, which is not to be enjoyed till the expiry of 99 years, reckoning compound interest at 3 p. c.

The value of the perpetuity at its commencement is $£300 \div \cdot 03 = £10000$; and the pres. val. of £10000 due in 99 years is $£10000 \div 1 \cdot 03^{99} = 10000 \div 18 \cdot 658866$, = £535·93825. *Ans.*

EXERCISES.

1. What is the present value of an annuity of £63 per ann., to continue 13 years, interest 5 p. c.?

Ans. £591. 15*s.* 10·82*d.*

2. Required the present value of 9 years' duration of a salary of £300, interest 4½ p. c.

Ans. £2180. 12*s.* 8·92*d.*

3. Compute the present worth of £150 per ann., for 42 years, at 3 p. c. *Ans.* £3555. 4*s.* 0·93*d.*

4. How much ready money should a person receive for the transfer of a pension of £35 per ann., to continue 7 years, interest 6 p. c.? *Ans.* £195. 7*s.* 8*d.*

5. Calculate the present worth of an annuity of £420, for 33 years, receivable at the end of every quarter; the interest, 4 p. c., being payable yearly.

Ans. £7735. 8*s.* 1·73*d.*

6. A lease of 50 acres, to continue 36 years, is offered at a rent of 30*s.* an acre; but the lessee agrees to pay a fine just now, in order to have the rent reduced to 16*s.* an acre. What ought the fine to be, reckoning compound interest at 6 p. c. per ann.?

Ans. £511. 14*s.* 8·29*d.*

7. A offers, for a 7 years' lease of certain profits, £150 of fine and £300 of yearly rent; B offers £450 fine, and £250 rent; C offers £750 fine and £200 rent; and D offers £1970 for the whole purchase. Required whose offer is highest, and the differences, computing at 3½ p. c.

Ans. A's offer exceeds B's by £5. 14*s.* 6·53*d.*;
B's exceeds C's by the same;
C's exceeds D's by £2. 18*s.* 2·11*d.*

8. What is the value of a freehold estate of £130 per ann., allowing 4 p. c. compound interest?

Ans. £3250.

9. What is the worth of a perpetuity of £76 a year, at 3½ p. c. per ann.? *Ans.* £2171. 8*s.* 6½*d.*

10. Required the value of a perpetuity of £90 a year, payable half-yearly; interest at 4½ p. c. per ann., payable yearly. *Ans.* £2022. 5*s.* 0·58*d.*

11. What is the present worth of £20 a year, to be obtained after 3 years, and to continue 20 years; interest at 4½ p. c.? *Ans.* £227. 19*s.* 6·29*d.*

12. Determine the present value of 60 years' possession of an estate of £100 a year, to begin at the expiration of 40 years; interest 3 p. c.

Ans. £848. 8*s.* 3·2*d.*

13. What annuity, to continue 20 years, is worth the present payment of £1246. 4*s.* 5*d.*, allowing the purchaser compound interest at 5 p. c.? *Ans.* £100.

14. What annuity will in 7 years redeem a loan of £200, interest 5 p. c.? *Ans.* £34. 11*s.* 3·35*d.*

15. When 12 years of a lease of 21 years were expired, a renewal for the same term was granted for £1000. Eight years of that lease are now expired; and it is required to determine what sum should be paid for a corresponding renewal of the lease, reckoning 5 p. c. comp. interest. *Ans.* £599. 18*s.* 6·4*d.*

16. What is the annual rent of an estate, the possession of which, for 15 years, is £400 better than the reversion of it for ever after the expiration of the 15 years, allowing comp. interest at 5 p. c.

Ans. £526. 15*s.* 9·58*d.*

MISCELLANEOUS QUESTIONS WITH SOLUTIONS.

1. A, B, and C, thus discoursed about their ages: Says A, "I am 30 years of age." Says B, "My age exceeds yours by a fourth of C's." "Then," says C, "in that case I am as old as you both." Required the ages of B and C.

Sol. The ages of A and B together amount to twice 30 added to one fourth of C's; and C's age amounts to the same; therefore, 60 is three fourths of C's; or C's age is four thirds of 60, viz. 80, and B's is 30 + 20, or 50. *Ans.*

2. A person lent a certain sum for 6½ years, at 5 p. c. per ann., simple interest, and found that if he had lent the same sum for 12 years and 9 months, at 4 p. c. per ann., he should have received £185 more of interest. What sum did he lend?

Sol. 6½ yrs. at 5 p. c. = 1 yr. at 32½ p. c.; also, 12¾ yrs. at 4 p. c. = 1 yr. at 51 p. c. Therefore, 51 hundredths — 32½ hundredths, of the required sum, is £185; or £185 ÷ 185 = £1000. *Ans.*

3. A party of men were employed to do a piece of work; and when they had wrought 24 days, and had finished one-third of the work, 15 men were added to their number, by which means the remainder of the work was accomplished in 12 days. How many men were at first employed?

Sol. The original workmen could have done the remaining two-thirds in 48 days; therefore, what 15 men did in 12 days was what the original party should have done in 36 days, or 36 : 12 :: 15 m. : 5 men. *Ans.*

4. A besieged garrison had a quantity of provisions sufficient for 6 weeks, if distributed at 10 ounces to each man per day; but having lost 630 men in a sally, the governor was enabled to increase the allowance to 12 ounces a day for 8 weeks. How many men were at first in the garrison?

Sol. The provision consisted of 420 oz. for each man of the original company, or 672 oz. for each man of the reduced company. Now, the surplus provision was 420 oz. for each of the 630 men lost; and this sufficed to give 252 oz. to each of the survivors. Therefore, 252 oz. : 420 oz. :: 630 m. : 1050 survivors; hence there were 1680 men at first. *Ans.*

5. A certain sum of money is to be distributed equally among a certain number of persons. The sum, it is observed, is 14*d.* too little to admit of each person receiving 9*d.*; but it is 18*d.* more than sufficient, if the share of each person were 5*d.* Determine the actual share of each.

Sol. The distribution of 4*d.* less to each requires (14*d.* + 18*d.*) 32*d.* less altogether. $32 \div 4 = 8$ persons; and 18*d.* + 8 times 5*d.* = 58*d.* the sum. Therefore $58 \div 8 = 7\frac{1}{2}d.$ the share of each. *Ans.*

6. A post in a stream has 4 feet in the ground, five-sixths of the remainder immersed, and one-eighth of the whole above water. Required its length.

Sol. Five-sixths of the remainder being immersed, one-sixth of that remainder is above water; therefore, one-sixth of that remainder equals one-eighth of the whole post, and five-sixths of that remainder equals five-eighths of the whole post. Accordingly, five-eighths + one-eighth of the post = 4 feet less than the whole length; or 4 feet is a quarter of the whole; so that the post is 16 feet long. *Ans.*

7. A waterman finds by experience, that with the advantage of a common tide, he can row down a river from A to B, which is 18 miles, in an hour and a half; and that to return from B to A, during the same tide, though he rows back along the shore, where the stream is only three-fifths as strong as in the middle, takes him just two hours and a quarter. Required the hourly velocity of the middle current.

Sol. He rows down at 12 miles an hour, and up at 8 miles an hour, thus making 4 miles less per hour in returning, because he has $1\frac{1}{2}$ less of the force of the middle current which previously aided him. Wherefore, $1\frac{1}{2}$ of vel. : 1 vel. :: 4 mi. : $2\frac{1}{2}$ mi. an hour, the velocity of the middle current. *Ans.*

8. If I was twice as old as my wife
On the day when we were wed;
And forty-three years of our married life—
All but a month—are fled;
And her age, as now compared with mine,
In ratio proves as seven to nine;—
Can any expert accountant say
What our ages were on our wedding-day?

Sol. The difference of their ages is at first $\frac{1}{2}$ of the man's, and afterwards $\frac{7}{9}$ of the man's; but the difference of their ages is always the same amount; therefore, the man's age at first is to his age subsequently, as $\frac{7}{9}$ to $\frac{1}{2}$, or 4 to 9; consequently, his age on his wedding-day is to its increase as 4 to 5; and 5 : 4 :: 42 yrs. 11 mo. : 34 yrs. 4 mo. the man's age on his wedding-day, &c. *Ans.*

9. Divide the number 90 into four such parts, that the first increased by 2, the second diminished by 2, the third multiplied by 2, and the fourth divided by 2, shall all yield the same result.

Sol. As 2 is added to and subtracted from the first and second parts, respectively, it is obvious that the 1st and 2nd together make twice *the common result;* also the 3rd part is $\frac{1}{2}$ *ditto*, and the 4th is twice *ditto*. The number 90 is, therefore, $2 + \frac{1}{2} + 2 = 4\frac{1}{2}$ times the common result; hence, that result is 20; and the four parts required are 18, 22, 10, and 40. *Ans.*

10. Bought a certain number of eggs at 2 a penny, and as many more at 3 a penny; and, by selling them all at the rate of 5 for 2*d.*, I lost 4*d.* How many eggs did I purchase?

Sol. Average buying price $(\frac{1}{2} + \frac{1}{3})d. \div 2$, or $\frac{5}{12}$ of a penny for each egg, $= 5d.$ the dozen. Now, as the selling price is $4\frac{1}{2}d.$ the dozen, there is a loss of $\frac{1}{2}d.$ on the dozen, amounting to a loss of 4*d.* on the whole. Accordingly,

$\frac{1}{2}d.$: 4*d.* :: 12 eggs : 240 eggs. *Ans.*

11. I have just returned from an hour's walk, having observed at the outset that the hour and minute hands of my watch pointed in one direction; and when I had walked a mile and a quarter, I found that they pointed in exactly opposite directions. At what rate per minute did I walk, supposing it uniform?

Sol. The minute hand had gained 6 hour spaces on the hour hand; but the minute hand passes over 12 hour spaces in the

time that the hour hand passes over 1; or, in 60 minutes, the minute hand gains 11 hour spaces; therefore,

11 sp. : 6 sp. :: 60 min. : $32\frac{8}{11}$ min. to walk $1\frac{1}{2}$ mi.; which is at the rate of $67\frac{1}{2}$ yds. per minute. *Ans.*

12. A farmer engaged a labourer at the commencement of the year, on condition of allowing him 16*d.* with his board for every day he wrought, and of charging him 9*d.* for his board every day he idled. Now, at the end of the year (313 days) it was found that the man should have received £11. 18*s.* 2*d.* altogether. How many days was the labourer idle?

Sol. If he had wrought every day, he should have received $16d. \times 313 = 5008d.$; but his actual receipt was 2150*d.* less, arising from a loss of 16*d.* + 9*d.* for every idle day; therefore, 25*d.* : 2150*d.* :: 1 da. : 86 idle days. *Ans.*

13. There is a fishing-rod composed of two pieces whose lengths have the proportion of 9 and 11; and 8 times the shorter piece, added to 5 times the longer, make 42 inches more than 6 times the length of the rod. Required its length.

Sol. The shorter piece is $\frac{9}{20}$, and the longer $\frac{11}{20}$, of the whole rod; therefore, 8 times the shorter + 5 times the longer, $= 6\frac{7}{20}$ times the length of the rod; so that 7 twentieths of the rod is 42 inches, or 20 twentieths = 120 in. = 10 ft. *Ans.*

14. Suppose there is a circular island, and that three men start together, at one point in the circumference, to go round it in one direction:—A performs one circuit in $6\frac{1}{2}$ hours, B does the same in $3\frac{1}{4}$ hours, and C the same in 3 hours. In what time should they be all together again at the starting point? and in what time should they first arrive together at one point?

Sol. They will be again together at the starting point in the time which is the least common multiple of $6\frac{1}{2}$, $3\frac{1}{4}$, and 3, = the least common multiple of the numerators 19, 19, and 3, divided by the greatest common measure of the denominators 3, 5, and 1, $= 57 \div 1$ or 57 hrs. *First Ans.*

Again: A performs $\frac{2}{13}$ of a circuit per hour, B $\frac{4}{19}$, and C $\frac{1}{3}$; therefore B gains on A $\frac{6}{19}$, and C gains on A $\frac{19}{57}$, per hour. Accordingly, B gains on A a whole circuit in $9\frac{1}{2}$ hrs., and C gains on A a whole circuit in $5\frac{7}{10}$ hrs.; or A and B are together for the first time in $9\frac{1}{2}$ hrs., and A and C ditto in $5\frac{7}{10}$ hrs. Consequently, A, B, and C are all together for the first time in the least comm. mult. of these times, viz. $28\frac{1}{2}$ hrs. *Second Ans.*

15. If a man weigh 192 lbs., and the specific gravity of his body be 1·2; how much cork (whose specific gravity is ·24) must be tied to him to make him swim?

Sol. $1\frac{1}{2}$ lb. of the man's body, $\frac{1}{2}$ lb. of cork, and 1 lb. of water, are equal volumes. Or, the weight of water equal in volume to

the man's body is $\frac{8}{9}$ of his weight, that equal in volume to the required quantity of cork is $4\frac{1}{2}$ times the weight of the cork, and that equal in volume to the compound body must be exactly the weight of the compound body, to make it swim.

We have to find therefore, by Alligation, what weight of cork at $4\frac{1}{2}$ volumes per lb. must be combined with 192 lbs. of another substance at $\frac{5}{8}$ of a volume per lb., that the compound may be at 1 volume per lb.

Accordingly, the weight of the cork is to the man's weight as $1 - \frac{8}{9} : 4\frac{1}{2} - 1$, or as 1 to 19;

$$19 : 1 :: 192 \text{ lbs.} : 10\tfrac{1}{19} \text{ lbs. cork. } \textit{Ans.}$$

16. If 3 peasants, or 4 soldiers, can excavate 21 cubic yards of earth for an intrenchment, in a day of $8\frac{1}{2}$ hours; in what time should 7 peasants and 9 soldiers excavate 64 cubic yards, supposing the soil, in the latter case, less favourable to expertness by one-fifth?

Sol. If 3 peasants = 4 soldiers, then 7 peasants = $9\frac{1}{3}$ soldiers; so that we have to compare the times of 4 soldiers and $9\frac{1}{3} + 9$ soldiers.

$$\left.\begin{array}{rcl} 18\frac{1}{3} \text{ sol.} & : & 4 \text{ sol.} \\ 21 \text{ yds.} & : & 64 \text{ yds.} \\ \frac{1}{4} & : & 1 \end{array}\right\} :: 8\tfrac{1}{2} \text{ ho.} : 6\tfrac{8}{9} \text{ ho. } \textit{Ans.}$$

17. King Charles II. laid the first stone of Chelsea Hospital on Feb. 16th, 1682. On what day of the week did the ceremony take place?

Sol. The days of the month recur on the same days of the week after every 28 years (*i. e.* 4 years × 7 intercalary days).

Now, from 1682 to 1850 is an interval of exact periods of 28 years. Therefore, we may first calculate the day of the week on which the 16th February, 1850, will fall. This is easily found to be Saturday. Therefore, in 1682, it should have fallen also on a Saturday:—But we have to move 1 day forward, on account of 1800 not being reckoned a leap year, and also 11 days forward, as so many were suppressed in September 1752 to introduce the New Style; and accordingly, 12 days beyond Saturday brings us to *Thursday*.* *Ans.*

18. Required each of three equal payments, due in 5, 8, and 9 months, respectively, if £22. 9*s.* $9\frac{1}{2}$*d.* paid at present, would discharge them all; interest 5 p. c.

Sol. The present worths of the common payment per £ are

* Another method of resolving such questions is given in Wilson's *Arithmetic* (Edinburgh, 1752). See also a *Table* for the purpose, published by Mr. Maynard of London.

$100 \div 102\frac{1}{2}$, $100 \div 103\frac{1}{2}$, and $100 \div 103\frac{3}{4}$; or $\frac{41}{42} + \frac{20}{21} + \frac{39}{40} = \frac{3239}{1680}$ of the common payment $= £22.\ 9s.\ 9\frac{1}{2}d.$

$367034 : 126077 :: £22.\ 9s.\ 9\frac{1}{2}d. : £7.\ 14s.\ 6{\cdot}05d.$ *Ans.*

19. Bought 15 cows and 130 sheep for £364. 11*s.* 8*d.*, and then sold the whole for £384. 8*s.* 4*d.*, at a profit of 7 per cent. on the cows, and 4 per cent. on the sheep. How much did I pay a head?

Sol.

Bought all for	£364	11*s.*	8*d.*
4 per cent.	14	11	8
	379	3	4
Sold all for	384	8	4

3 p. c. on the cows $=$ £5 5 0

£3 : £5$\frac{1}{4}$ :: £100 : £175, cost of all the cows;

175 $\div$ 15 $=$ £11. 13*s.* 4*d.* cost of a cow } *Ans.*
£1. 9*s.* 2*d.* ditto sheep }

20. If 36 men can do 52$\frac{1}{2}$ measures of work in 4 days, working 7 hours a day; what uniform length of day will 40 men require, so that 39$\frac{1}{4}$ measures may be done in 3$\frac{1}{2}$ days, supposing that any 5 of the latter party can do in 7 hours what any 8 of the former party can do in 5$\frac{1}{4}$ hours, and that from the latter party 3 men are to be withdrawn after 2$\frac{1}{2}$ days' work?

Sol.

40 men	36 men	
52$\frac{1}{2}$ mea.	39$\frac{1}{4}$ mea.	:: 28 ho. : 15·7 ho.
8 men	5 men	
5$\frac{1}{4}$ ho.	7 ho.	

The 40 superior workmen could do the work in 15·7 hours; or one of these men would take 628 hours. But the work is required to be performed by 2$\frac{1}{2}$ days of 40 such men + $\frac{5}{8}$ day of 37 such men, $=$ 130$\frac{5}{8}$ days of 1 man; therefore 130$\frac{5}{8}$ days must be made equal to 628 hours, which determines the uniform length of day to be 4 ho. 48 min. *Ans.*

21. If £146. 9*s.* 3*d.* is to be lent in 3 separate sums, at the respective rates of 5, 3, and 4$\frac{1}{2}$, per cent. per annum, and for 4, 6, and 8 months, respectively; what must each loan be, that the interest payments may be equal? Also; what must each loan be, that the amounts, including interest, may be equal?

Sol. The first loan $\times$ $\frac{4}{12}$ p. c., the second $\times$ $\frac{6}{12}$ p. c. and the third $\times$ 3 p. c., are all equal products; therefore the three payments are as $\frac{4}{12}$, $\frac{6}{12}$ and $\frac{1}{1}$, or as 9, 10, and 5, to make the interests equal;

24 : 9 :: £146. 9*s.* 3*d.* : £54. 18*s.* 5$\frac{1}{2}$*d.* first loan, 10 ninths of which is the second, and 5 ninths of it the third loan. *Ans.*

Secondly, To make the amounts equal, we have $\frac{1}{20}$ of the first

loan $= 1\frac{6}{200}$ of the second $= 1\frac{6}{100}$ of the third; or, the loans are as $\frac{200}{206}$, $\frac{100}{106}$, and $\frac{100}{103}$, or as 62727, 62830, and 61915. Therefore, dividing £146. 9*s*. 3*d*. in these proportions, we obtain £49. 0*s*. $1\frac{5}{12}$*d*. the first loan, &c. *Ans.*

22. A can do a work in 10 days, B in 9 days, C in 12 days. They all begin it together; but C alone continues till the work is finished; A leaving it $3\frac{3}{4}$, and B $2\frac{2}{3}$ days, before its completion. In what time is the work performed?

Sol. $\frac{1}{10} + \frac{1}{9} + \frac{1}{12} = \frac{37}{180}$ of the work per day by the three while they work together;

$\frac{3\frac{3}{4}}{10} + \frac{2\frac{2}{3}}{9} = \frac{14}{120}$ of the work; which subtracted from $\frac{37}{180} \times$ C's time, gives the whole amount of work performed during C's time. Accordingly, $\frac{37}{180} \times$ C's time $= 1\frac{14}{120}$; and therefore C's time is $5\frac{1}{37}$ days. *Ans.*

23. A offers to sell me a quantity of spirits at 25*s*. 10*d*. a gallon, with a certain term of credit, or at 26*s*. 3*d*., with 2 months longer credit. For what ready money did he purchase the article 2 months ago, if he is now demanding a profit of $12\frac{1}{2}$ p. c. on his full outlay, and reckoning the use of money to be worth 10 p. c. per ann.?

Sol. 26*s*. 3*d*. — 25*s*. 10*d*. = 5*d*. = 2 mths. int. or $1\frac{2}{3}$ p. c. on the present worth; therefore, £$1\frac{2}{3}$: 5*d*. :: £100 : 25*s*. the present worth. $112\frac{1}{2}$: 100 :: 25*s*. : 22*s*. $2\frac{2}{9}$*d*. prime cost at present; $101\frac{2}{3}$: 100 :: 22*s*. $2\frac{2}{9}$*d*. : 21*s*. $10\frac{31}{51}$*d*. prime cost 2 mths. ago. *Ans.*

24. The values of diamonds of equal quality are as the squares of their weights. If, therefore, two diamonds, weighing 10 and 21 grains respectively, are worth £270. 10*s*.; what is the weight of a diamond worth £70?

Sol. $10^2 + 21^2$: 10^2 :: £270. 10*s*. : £50, the value of a diamond weighing 10 grains; therefore,

£50 : £70 :: 10^2 : 140; $\sqrt{\ } = 11{\cdot}832$ grs. *Ans.*

25. If the square of the value of rubies varies as the cube of their weight; what is the weight of a ruby worth two others of 4 and 9 grains respectively?

Sol. The squares of the values are as the cubes of the weights, or the values are as the square roots of the cubes of the weights; therefore the comparative values of the rubies of 4 and 9 grains are $\sqrt{4^3}$ and $\sqrt{9^3}$, or 8 and 27, and the proportionate value of both is 35; but one ruby worth 35 will weigh $\sqrt[3]{35^2} = 10{\cdot}7$ grs. *Ans.*

26. A and B join stocks in trade, and A's stock continues $\frac{1}{3}$ of the time of B's. When they come to share the profits, it is found that, for every £56 B gets, A receives £15, and that A's

stock and gain together amount to £495. Shortly afterwards, they again join their original stocks for the same times, and admit C a third partner, with a stock equal to the difference of their own, and for a period half as long as A's. The total gain turns out the same as before; but A's share of it is now £16. 13*s.* 4*d.* less than formerly. Required the stocks of A and B.

Sol. First; A's time is to B's as 5 : 8; A's gain is to B's as 15 : 56; and therefore A's stock is to B's as 3 : 7.

Secondly; A, B, and C's stocks are as 3, 7, and 4; their times as 5, 8, and $2\frac{1}{2}$; and therefore their gains as 15, 56, and 10.

In the first partnership, A has $\frac{15}{71}$, and in the second, $\frac{15}{81}$, of the gain; these are as 81 and 71; diff. 10; therefore,

10 : 81 :: £16$\frac{1}{2}$: £135 A's first gain;

495

3 : 7 :: 360 : £360 A's stock, / 840 B's ditto. } *Ans.*

27. A has a cargo of corn, which he wishes to barter with me for a quantity of linen, at the rate of 4 bushels for every 3 yards. The yard of linen cost me 29*s.* less than the quarter of corn cost him; but, altogether, the cloth cost me one-fifth more than the grain cost him. Now, I find that if I could dispose of the linen, even at prime cost, with 9 months' credit, I should obtain, on the principles of true discount and legal interest, 6*s.* 7$\frac{1}{2}$*d.* more than by accepting A's exchange, though I might sell the corn for ready money, at 15 per cent. above what it cost him. Required the quantity of linen.

Sol. The number of quarters of corn is to the number of yards as 1 : 6; and the whole cost of the corn is to ditto of cloth as 5 : 6. Therefore, the cost per quarter is to the cost per yard as $5 \div 1 : 6 \div 6$, or 5 : 1.

5 — 1 : 1 :: 29*s.* : 7*s.* 3*d.* prime cost per yard.

Now, the proportionate present values of the selling amounts of the cloth and the grain are $\frac{100}{103\frac{3}{4}} \times 6$ and $\frac{115}{100} \times 5$, or 1920 and 1909; therefore, the present amount of the linen is 1160*s.*, for, 11 : 1920 :: 6*s.* 7$\frac{1}{2}$*d.* : 1160*s.*

9 months' interest 43$\frac{1}{2}$

Prime cost of the linen, 1203$\frac{1}{2}$*s.*

which, at 7*s.* 3*d.* the yard, requires 166 yards. *Ans.*

28. Suppose the grass in a meadow to be of uniform quality and growth, and that, when the whole pasture is equivalent to 12 weeks' growth, 48 sheep being admitted would crop it completely down in 4 wks.; how long would it take 28 sheep to do the same?

Sol. If 48 sheep eat (12 + 4) or 16 weeks' growth in 4 weeks,

28 sheep would eat 9½ weeks' growth in 4 weeks, in which case the original grass would be 5½ weeks' growth; therefore, to suit the true original grass,

5½ wks. gro. : 12 wks. gro. :: 4 wks. : 9 wks. *Ans.*

29. If I engage 10 workmen for 4 weeks, or 6 workmen for 12 weeks, I can exactly defray their wages, at a uniform rate, by means of the interest now accumulated on a certain sum of money together with the interest which arises during the particular term of engagement. How many workmen can I engage for 9 weeks on the same principle?

Sol. The original interest + 4 weeks' interest pays 10 for 4 wks., = 1 man for 40 wks.

The original interest + 12 weeks' interest pays 6 men for 12 wks., = 1 man for 72 wks.

The difference of these two amounts shows that 8 wks. interest would pay the wages of 1 man for 32 wks.; or 10 weeks' interest would pay the wages of 1 man for 40 wks. Now, comparing this with the first statement, we observe that as the original interest + 4 weeks' interest would pay 1 man for 40 wks., and as 10 wks. interest would pay the same, therefore, the original sum of interest must be 6 wks. interest.

And the question requires to find how many men could be paid for 9 wks. by means of the original interest + 9 wks. interest, i. e. by 15 wks. interest.

$$\left.\begin{array}{l} 10 \text{ wks. int.} : 15 \text{ wks. int.} \\ 9 \text{ wks.} \quad\quad : 40 \text{ wks.} \end{array}\right\} :: 1 \text{ man} : 6\tfrac{2}{3} \text{ m. } \textit{Ans.}$$

30. Suppose a cistern is supplied by a regular and continual influx of water, and that, when it contains a certain quantity, 25 equally discharging pipes, being set open, would drain it in 96 minutes, or that 40 such pipes would drain it in 12 minutes; how soon would 32 such pipes drain it?

Sol. The original quantity + 96 minutes' supply is drained by 25 pipes in 96 min., = 1 pipe in 2400 min.

The original quantity + 12 minutes' supply is drained by 40 pipes in 12 min., = 1 pipe in 480 min.

Therefore, we find by subtraction, that 84 min. sup. would be drained by 1 pipe in 1920 min.; or 21 min. sup. by 1 pipe in 480 min. 21 — 12 = 9 minutes' supply is the original quantity.

And the question requires to find how soon 32 pipes would drain the original quantity + the supply during the time sought. Now, 21 min. sup. would be drained by 1 pipe in 480 min., or by 32 pipes in 15 min.; in which case the original quantity would be 6 min. sup.; therefore, to suit the true original quantity,

6 min. sup. : 9 min. sup. :: 15 min. : 22½ min. *Ans.*

Miscellaneous Exercises.

I. Divide £917. 19*s*. 6*d*. among A, B, and C, so that A may have £320. 7*s*. 4*d*. more than B, and B £63. 15*s*. 10*d*. more than C.

Ans. A £540. 16*s*. 8*d*.; &c.

II. A and B together have £87, and C has £14 more than A and £30 less than B. What sum has each? *Ans.* A £21. 10*s*., &c.

III. The population of a parish is 3496; what was its population 5 years ago, the total increase during that period having been about 7·37 per cent.?

Ans. 3256.

IV. Divide 9*s*. 3*d*. among Robert, William, and Henry, giving R. 2*d*. as often as you give W. 1½*d*., and giving H. 1½*d*. as often as you give W. 2*d*.

Ans. R. 4*s*., W. 3*s*.; &c.

V. A workman is to receive a certain remuneration, if he produces 138 measures of work in 13 days. At the end of 4 days, he has finished 42½ measures. In what proportion may his rate of working now be altered, so as just to secure the remuneration?

Ans. Reduced as 765 : 764.

VI. What vulgar fraction divided by its reciprocal becomes $\frac{11}{14}$? *Ans.* $\frac{121}{196}$ nearly.

VII. A has 46 crowns and a sovereign, B has 46 sovereigns and a crown; what sum must A give to B, that the latter may then have 4 times as much as the former? *Ans.* 3 crowns.

VIII. A and B set out from two towns 59 miles distant, and travel directly towards each other; A goes 3½ miles, B 2¾ miles an hour; but B having set out an hour later than A, it is required to find how far B shall have travelled when they meet. *Ans.* 24 mi.

IX. Divide 46 into two such parts, that the one increased by 5 may equal the other diminished by 6.

Ans. 17½; 28½.

X. E requires 12 da. 4 hrs. to do a work which F

can accomplish in 10 da. 7 hrs. 30 min., and G in 11 da. 5 hrs. 20 min. In what time may the work be performed by their united exertions, allowing 8 hours for a day's work? *Ans.* 3 da. 7 hrs. 6⅔ min.

XI. A mixture is composed of 83 pints of rum and 9 of water; how much rum must be added, that 25 pints of the mixture may contain 2 pints of water?

Ans. 20⅔ pts.

XII. Find two numbers, the greater of which shall be to the less as their sum is to 72, and as their difference is to 27. *Ans.* 108·9; 49·5.

XIII. If the French gold piece of 20 francs weigh 97¾ grains Troy, and 1 oz. Troy of standard gold be worth £3. 17*s.* 10½*d.*; how many francs may be reckoned equivalent to £1 sterling? *Ans.* 25 frs. 22 cts.

XIV. I have sold, at a loss, a neat cottage and grounds,
For the sum of One thousand and ninety-five pounds;
Had they sold for Eleven hundred guineas, my gain
Would have proved to be 3 times the loss I sustain.
Then, tell me the sum which the property cost;
And how much per cent. by its sale I have lost.

Ans. Cost £1110; Loss $1\cdot\dot{3}5\dot{1}$ p. c.

XV. A company of tradesmen, working 8 hours a day for 15 days, can accomplish a work which another company can perform in 16 days working 7 hours a day. For how many hours a day should these parties be engaged together, that the work may be done in 6 days?

Ans. 9⅓ hrs.

XVI. By a loss of 7 per cent. my money was reduced to 20 crowns, 3 shill. and 4 pence. To what should my money have been reduced, had my loss been only 4½ per cent.? *Ans.* £5. 6*s.* 1½*d.*

XVII. There are two porter vats of such dimensions that ⅘ of the first contains 96 gallons less than ⅔ of the second, and ⅝ of the second contains as much as ½ of the first. What does each vessel hold?

Ans. 720 g.; 512 g.

XVIII. Find a number, which added to itself, multiplied by the sum, subtracted from the product, and dividing the remainder, the quotient may be 13.

Ans. 7.

XIX. L, M, and N, start at the same time and place, to travel in the same direction round an island 60 miles in circuit. L goes 5·4 miles per day, M 16·2 miles, and N 34·2 miles. How soon will they all come together again? 16⅔ da.

XX. What sum of money is such, that if $\frac{1}{11}$ of it be distributed among four persons, and the remainder improved at 5 p. c. simple interest for 12 years, and then $\frac{1}{11}$ of the amount be distributed as before, there shall be £102 left? *Ans.* £1800.

XXI. A grocer sells tea at 5*s.* 3*d.*, coffee at 2*s.* 3*d.*, and sugar at 6*d.*, per lb.; and having one day sold 15 lbs. of coffee more than of tea, he found that the quantities of the three articles sold that day amounted to equal sums of money. Required those quantities.

Ans. T. 11¼ lbs. C. 26¼ lbs. S. 118$\frac{1}{2}$ lbs.

XXII. From a cask, containing 96 gallons of spirits, a certain quantity was drawn, and the cask filled up with water; then the same quantity of the mixture was drawn, and the cask filled up with water; and when this was done for the fourth time, there remained only 30$\frac{3}{8}$ gallons of spirits in the cask. What quantity was drawn off each time? *Ans.* 24 gall.

XXIII. There is a cistern whose length is 2½ times its depth, and its breadth ¾ of its length, and containing 5553·6615 imperial gallons. What is the depth of the cistern? *Ans.* 5¾ feet.

XXIV. A woman sold 90 apples, some at the rate of 3 for 2½*d.*, and the rest at 8 for 6½*d.*; and she found that those sold at the latter price brought her just 4*d.* more than the others did. How many were sold at each rate? *Ans.* 42 at 3 for 2½*d.*; &c.

XXV. To how much would 1 penny amount in 400 years, if improved at 5 p. c. per ann. compound interest?
Ans. £1245972. 5*s*. 11½*d*.

XXVI. A boatman rows from Cambridge to Ely, a distance of 20 miles, and back again, in 10 hours; and he finds that he can row 2 miles against the stream in the same time that he can row 3 miles with it. How many miles an hour does he row down and up?
Ans. 5 with; 3½ against.

XXVII. The sum of £247 was divided into equal numbers, of guineas and crowns respectively; and the guineas being improved at 4 p. c. per ann. simple interest, and the crowns at 5½ p. c., each for the same period, the whole amount turned out £299. 19*s*. 3*d*. Required the time. *Ans.* 5 yrs.

XXVIII. The discount on a promissory note of £100 amounted to £1. 13*s*. 4*d*.; and the interest made by the banker was £5·0647 per cent. per ann. Find the interval at the end of which the note was payable.
Ans. 4 mths!

XXIX. A, B, and C, trade with a joint stock of £1534. A's stock is £456, and his time 7 months; B's stock is £546, and his gain £132. 12*s*.; C's gain is £108. 16*s*., and his time 8 months. Required A's gain and B's time. *Ans.* £81. 12*s*.; 9½ mths.

XXX. A number of labourers, men and women, were paid 2 guineas for reaping a field; each man received 2*s*. 3*d*. a day, and each woman 1*s*. 6*d*. Had there been 2 men and 1 women more, the two parties should have received equal amounts. How many men and how many women were there? *Ans.* 12 m.; 10 w.

XXXI. A and B had each a certain number of sovereigns. Said A to B, 'If you give me 3 of your sovereigns, I shall then have as many as yourself.' 'But,' said B, 'if I had 7 of your sovereigns, you should then have only a third of my number.' How many sovereigns had each? *Ans.* A 17; B 23.

XXXII. I once paid my house-rent of fifty-three pounds
With money consisting of sovereigns and crowns;
The pieces in all were a hundred and seven:—
Can you tell me how many of each kind were given?
Ans. 35 sov. 72 crs.

XXXIII. Divide £100 into two sums, so that the true discount of the one for 4 years at $3\frac{1}{2}$ per cent. per ann. may equal the true discount of the other for 6 years at $2\frac{1}{2}$ per cent. per ann. *Ans.* £51$\frac{4}{11}$; £48$\frac{7}{11}$.

XXXIV. M and N by their united exertions can finish a certain work in $32\frac{1}{7}$ days; and M can do as much of it in 3 days as N can do in 5. In what time could each by himself perform the whole work?
Ans. M 51$\frac{3}{7}$ da.; N 85$\frac{5}{7}$ da.

XXXV. Mercury dilates in volume by 1 measure on 5550 for each degree of the French, or Centigrade, thermometer. What, in English cubic feet, will the volume of 2·15 French litres of mercury become, in passing from the temperature 50° Fahrenheit's to 36° Réaumur's scale, the litre measuring 61·027 cubic inches?* *Ans.* ·0764094 cub. ft.

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* In the Centigrade scale of heat the Freezing Point is 0°, and the Boiling Point 100°; in Fahrenheit's scale the F. P. is 32°, and the B. P. 212°; in Réaumur's the F. P. is 0°, and the B. P. 80°. Consequently, 100° C. = 180° F. = 80° R.

The following elementary problems may serve as Exercises in Simple Proportion:—

Required the degrees of Fahrenheit corresponding to 60° of the Centigrade. *Ans.* 140° Fahr.

When Fahrenheit's thermometer stands at 87°, at what will Réaumur's stand? *Ans.* 24$\frac{4}{9}$ deg.

What will the Centigrade indicate when Réaumur's scale indicates 18°? *Ans.* 22$\frac{1}{2}$ deg.

To what degrees of Réaumur correspond 8° below zero on the Centigrade? *Ans.* 6$\frac{2}{5}$ deg. below zero.

Convert 50° Fahr. into degrees of the Centigrade. *Ans.* 10° C.

Reduce 23°.Fahr. to degrees of the Centigrade.
Ans. 5° below zero.

Convert 3° Centigrade below zero into degrees of Fahrenheit.
Ans. 26·6° Fahr.

XXXVI. D, E, F, and G, are to be paid £2. 14*s.* for finishing a certain work. D could do it all himself in 16 days, E in 24, F in 36, and G in 48; but E begins some time after D, and F and G begin, both together, some time after E. D receives 26*s.* 9¾*d.* more than F, and E and G together get 16*s.* 8¼*d.* In what time is the whole work completed? And how long does D work before E begins, and E before F and G begin?

Ans. The whole work is completed in 9½ da.; D 3¾ da. before E; E 2½ da. before F and G.

XXXVII. Divide the number 113 into two parts, so that 7 times the greater added to 13 times the less shall make 1019. *Ans.* 75 and 38.

XXXVIII. A offers to purchase from me 12 cwt. of sugar at 55*s.* per cwt. cash. B proposes to take the sugar at 57*s.* 6*d.*, giving tea in exchange at 4*s.* 9½*d.* a lb., which I can sell at 5*s.* Now, suppose that, on an average, I can have cash for all the tea in 6 months, and that the use of money is worth 10 p. c. per ann.; which of the offers is the more advantageous for me?

Ans. B's by £1. 5*s.* 8½*d.*

XXXIX. Three men, C, D, and E, start at the same time, and from the same point in the circumference of a circle 71 miles round; C goes 50 miles a day, and E 40, in opposite directions round the island; and D goes along the island's diameter at the rate of 35 miles a day. How soon will the three be again at the point of outset, reckoning the ratio of the diameter of a circle to its circumference to be 113 : 355? *Ans.* 3209½ da.

XL. Two Parisians trade with equal stocks:—The first gains 10 per cent. every 4 months, and immediately applies that profit to the enlargement of his capital; the second gains 15 per cent. every 6 months, and does likewise; and, at the end of a year, the first has gained 408 francs more than the second. Required the original capital of each. *Ans.* 48000 francs.

XLI. I have gained 10 per cent. by retailing a quantity of goods, purchased at 6*d.* per lb. *net*, being allowed a certain proportion of tare, and the usual tret of $\frac{1}{18}$. Had the goods, however, cost me 6*d.* per lb. *gross*, I should then have lost 10 per cent. in the retail. Required how much per cwt. I was allowed for tare.

Ans. $16\frac{1}{2}\frac{8}{7}\frac{4}{7}$ lbs.

XLII. Paid 7*s.* 8*d.* for a basket of lemons and oranges, the former at $2\frac{1}{2}d.$, and the latter at 1*d.* I then sold the whole together at $1\frac{1}{4}d.$, and gained 6*d.* How many had I of each kind? *Ans.* 24 lem. 32 or.

XLIII. What is the shortest distance that can be travelled in an exact number of seconds, whether the rate per second be $6\frac{7}{8}$, $4\frac{9}{17}$, $2\frac{1}{11}$, or $3\frac{1}{11}\frac{4}{5}$ yards?

Ans. $1347\frac{1}{2}$ yds.

XLIV. I gave £3. 10*s.* for two dozen of wine, at different rates per dozen; and by selling one kind at an advance of 25 per cent., and the other at a reduction of 15 per cent., I obtained a uniform price for both. What did each dozen cost me? *Ans.* 28*s.* 4*d.*; 41*s.* 8*d.*

XLV. There are three similar rectangular vessels, 3, $3\frac{1}{2}$, and 4 feet deep, respectively; and the capacity of the largest is $23\frac{1}{2}$ cubic feet more than half the contents of both the others. Required how many imperial gallons each vessel can contain.

Ans. 134·6, 213·8, and 319·1 gall.

XLVI. I was driving two score of geese to town,
When I met with farmer A;—
'I suppose,' said he, 'you 'll demand a crown
For each of your geese to-day.'
Said I, 'If I sell them, all but eight,
At a profit of tenpence a-piece,
I 'll thus obtain an amount as great
As I gave for all the geese;
Now, farmer A, what I 've told may lead
To determine what each cost me;
So try to disclose it, and if you succeed,
I shall make you a present of three.'

Ans. 3*s.* 4*d.*

XLVII. A. B. of Glasgow orders from Lisbon 390 bags of cotton wool, weighing net 475 cwt. 2 qrs. 10 lbs., at 420 reis per lb.; agent's commission $2\frac{1}{2}$ per cent.; exchange 65*d.* per milrei; other charges 4*s.* 8*d.* per cwt.; insurance $5\frac{1}{2}$ per cent.; freight 7 guineas a ton. He keeps it on hand 2 months, warehouse rent 2*d.* a bag per month, and then sells the whole at 3*s.* per lb., for bills at 3 months' date, which he discounts. Required his gain or loss. *Ans.* Gain £982. 11*s.* $7\frac{1}{2}$*d.*

XLVIII. I bought a certain quantity of silk velvet at 18*s.* a yard, and sold it at an advance of 15 per cent. Again, I bought a quantity greater than the former by 4 yards, but at 3*s.* less per yard, and by selling it at a loss of 10 per cent., I received for it the same amount as for the former quantity. Required each quantity.

Ans. $7\frac{1}{2}$ and $11\frac{1}{2}$ yds.

XLIX. What sum would yield £100 of compound interest, in 100 years, at $2\frac{1}{2}$ per cent. per ann.?

Ans. £9. 4*s.* 11·4*d.*

L. A railway train, moving with uniform speed, is met by an engine and tender, 25 feet long, running 27 miles an hour. The engine and tender pass the train in $4\frac{1}{2}$ seconds after meeting it; and, returning shortly afterwards, at the same rate, repass the train in $20\frac{1}{2}$ seconds after overtaking it. Required the length of the train. *Ans.* 250 feet.

LI. Two labourers, John and William, are to dig a trench from P to Q, in length 324 yards. John commences at P, and digs 20 yards the first day, 18 the second, 16 the third, and so on; William begins at Q, at the same time when John commenced, and finishes 4 yards the first day, 9 the second, 14 the third, &c. In what time will the trench be completed? *Ans.* 9 days.

LII. There are two vessels, S and T, each containing a mixture of water and wine, S in the proportion of 2 : 3, and T in the proportion of 3 : 7. Determine what quantity must be taken from each, in order to form

a third mixture, which shall consist of 5 gallons of water and 11 of wine.

Ans. 2 gall. from S; 14 from T.

LIII. A cistern 25 feet long, 15 broad, and 10 deep, is to be altered and made 4 times as large, so that its depth may be $7\frac{1}{2}$ feet, and its sides as 3 : 5. Required the length and breadth.

Ans. 57·735 and 34·641 ft.

LIV. A sells to B 245 yards of cloth at 3*s.* $9\frac{1}{2}$*d.*, and at the same time B sells to A $91\frac{7}{8}$ gallons of spirits at 10*s.* $3\frac{1}{2}$*d.*; and to balance accounts, B allows A 3 months longer credit than he asks from A. Required the respective terms of credit, reckoning true discount and legal interest.

Ans. A gives 4 mths., B 7 mths.

LV. To complete a certain work, A takes 7 fifths of the time of B and C together, B twice as long as A and C together, and C thrice as long as A and B together; and A, B, and C, by their united efforts, can accomplish it in 5 days. Required the time in which each could do it alone. *Ans.* A 12, B 15, C 20 days.

LVI. I have bought 63 dozen of sherry for £78. 15*s.*, and intend to reserve part for private use, and to sell the remainder at 26*s.* 9*d.* a dozen. What quantity must I sell, that my sole pecuniary advantage may consist in having my own reserved quantity at 17*s.* $6\frac{1}{2}$*d.* a dozen? *Ans.* 51 doz.

LVII. A person performs a journey at a certain rate. Had he travelled $1\frac{1}{4}$ mile an hour quicker, he should have accomplished the journey in $\frac{4}{7}$ of the time; but, had he travelled $\frac{1}{4}$ of a mile per hour slower, he should have been $2\frac{1}{2}$ hours longer on the road. Find the distance travelled. *Ans.* $8\frac{1}{4}$ mi.

LVIII. A and B have equal incomes. A spends yearly £10 less than $\frac{4}{5}$ of his income, and B saves £17 every six months; also, what A saves is $\frac{4}{5}$ of what B spends, per annum. Find the income of each.

Ans. £122. 10*s.*

LIX. Some few years ago, in the county of Fife,
A wealthy old miser departed this life,
Leaving six sons behind to inherit his store,
Ninety-one hundred sovereigns and seventy pounds more.
Now, the question I 'd have you to answer is this—
What was left to each son at the father's decease?
It having been by the testator provided
That the whole of his fortune should thus be divided:—
Let George, who is eldest, a fixed sum obtain,
And William *one-third* of the pounds that remain;
One-third of the rest give to Harry, and then
Let *three-eighths* of what 's left be apportioned to Ben.
When *five-ninths* of the residue Tom shall receive,
To Charles whatever remains he must leave;
And this last remainder (if right the division)
Will prove just *one-fifth* of the eldest's provision.
Ans. George £3500; &c.

LX. A. B. receives the amount of an annual pension which has been forborn 15 years, at 5 p. c. per ann., compound interest. C. D. receives at the same time the sum of £778. 9*s*. 6*d*. with the arrears of compound interest on that sum for the same period at the same rate. They then invest their amounts in a joint speculation, A. B. for 9 months, and C. D. for 12 months, and thereby each party gains £50. Required A. B.'s yearly pension. *Ans.* £100.

LXI. I planned a piece of ground into a nursery, and the plants at the end of 6 years sold for £156, the average yearly expense being £5. Had the same ground been sown in grain, my clear annual profit, after deducting all expenses, would have been £20. Now, supposing that, in each case, the expenses were paid at the end of each year, and reckoning compound interest at 5 p. c. per ann. payable yearly,—which of these methods would be the more advantageous at the end of the six years? *Ans.* Grain, by £14. 0*s*. 11½*d*.

LXII. A person set out at a uniform rate of travelling from Genoa for Pisa; but, after going $\frac{1}{7}$ of his journey, he was apprised that banditti were in the way, and came back to Genoa, accomplishing the return in $4\frac{1}{7}$ hours. Next day, he resumed his journey, and walking uniformly 10 hours a day, and $1\frac{1}{5}$ furlong per hour slower than before, reached Pisa in $3\frac{1}{5}$ days. Required the distance between Genoa and Pisa.

Ans. 100 miles.

LXIII. The powers of three agents are, respectively, 14, 15, and 16; and by their joint operation a certain effect is accomplished in $3\frac{1}{3}$ days. In what time may each agent, operating singly, produce 10 times the effect? *Ans.* 1st 125 da., 2nd $116\frac{2}{3}$ da., 3rd $109\frac{2}{3}$ da.

LXIV. There is a ditch of uniform width and depth, the length of which contains just as many lineal feet as the area of its end contains square feet; and the earth dug from it forms a parapet of 17000 cubic feet. Required the length of the ditch, supposing the soil to be increased $\frac{1}{12}$ in volume by removal. *Ans.* 125·2689 ft.

LXV. A and B are partners in trade. A invests $\frac{5}{17}$ of the joint capital, for $10\frac{1}{2}$ months, and B receives $\frac{6}{17}$ of the gain. Required B's time. *Ans.* 8 mths.

LXVI. John begins a journey from N to P at the same time that James sets out from P for N. They meet in 3 hrs. $51\frac{1}{2}$ min., James's rate of travelling being $\frac{14}{15}$ of John's. How many hours does each person take to the whole journey?

Ans. John 7 hrs. $11\frac{1}{17}$ min.; James 8 hrs. $17\frac{7}{17}$ min.

LXVII. A father's age at present is 12 times the 5th of his son's, and 8 times the 3rd of his daughter's; and if the three persons live $11\frac{1}{2}$ years longer, the father's age will then be the sum of the son and daughter's ages. Find the age of each.

Ans. 54, $22\frac{1}{2}$, and $20\frac{1}{2}$ yrs.

LXVIII. M and N are partners in trade, M's stock being £240, and his time $\frac{7}{8}$ of N's; also, M's stock

and gain amount to £297, and N's to £445. Find the whole gain. *Ans.* £152.

LXIX. What uniform payment, at the termination of each of the next 5 years, will discharge a debt of 8841·01 francs due 1 year hence, reckoning interest at 5 per cent. per ann.? *Ans.* 1944·81 frs.

LXX. A gentleman travelling borrowed at one place as much money as he had then about him, and out of the whole spent £53. 19*s.* 11¼*d.*; at another place he borrowed as much money as he had now about him, and out of the whole spent the same sum as formerly. Having repeated this process 18 times more, he at the last found himself possessed of £1146. 5*s.* 3¼*d.* How much money had he about him at first?

Ans. £54.

LXXI. A sets out from Dieppe towards Paris, and B at the same time from Paris towards Dieppe. When they meet on the road, it appears that A has gone 17·95 miles more than B, and that, if they continue at their previous rates of travelling, A shall reach Paris in 11 hours, and B reach Dieppe in 21 hours. Required the rates of travelling, and the distance between the places.

Ans. A 4·275 mi., B 3·094 mi. an hr.; 112 mi.

LXXII. A composition of 110 lbs. is made of copper and tin. Required the quantity of each ingredient, estimating the specific gravity of the copper to be 9, of the tin 7·2, and of the compound 8·8.

Ans. 100 lbs. cop.; 10 lbs. tin.

LXXIII. A and B are joint stock partners, their periods of investment added together being 8 months; also, their gains, which are as the squares of their stocks, are £85. 15*s.* and £141. 15*s.*, respectively. Required the times. *Ans.* 3½ and 4½ mths.

LXXIV. A farmer kept a servant for every 50 acres he possessed, and on taking a lease of 110 acres more, he engaged 4 additional servants, after which he had a

servant for every 41 acres. How many acres had he at first? *Ans.* 300 ac.

LXXV. Find three numbers such that the first may be $\frac{8}{10}$ of the sum of the second and third, the third equal to the sum of the first and second, and the cube of their sum greater than the sum of their cubes by 15486·9. *Ans.* 6·088683; 7·103463; &c.

LXXVI. There are two numbers having the ratio of 1 to 3; and the square root of the cube of the less is to the cube root of the square of the greater as 5 to 2. Find the numbers. *Ans.* Less 7·23144; &c.

LXXVII. In two contiguous parishes, A and B, there are 19470 inhabitants altogether. In A the proportion of males to females is 4 : 5; and in B the proportion is 5 : 7; also, A contains 635 females more than B. Required the population of each parish.

Ans. A 10530 inh.

LXXVIII. A merchant in London holds two drafts, on Cadiz and Madrid respectively, the contents of which are such that 3 months' interest of the former would be $1\frac{1}{2}$ per cent. better than 5 months' interest of the latter. The sum of the drafts in Spanish plate is 7096 reals $31\frac{1}{2}$ maravedis, the current rate of exchange in Madrid being about $3\frac{8}{10}$ per cent. higher than at Cadiz, and the average exchange for both drafts 38·4644*d.* sterling per peso. Required the rates of exchange separately.

Ans. On Madrid $39\frac{1}{2}d.$ per peso; on Cadiz 38*d.*

LXXIX. If a person possessed of £1000 improve his money at 5 p. c. per ann. compound interest, what annual expenditure for his maintenance will allow his money to last just 14 years? *Ans.* £101. 0*s.* $5\frac{1}{2}d.$

LXXX. Suppose there is a meadow, of 8 acres, in which the grass grows uniformly, and that 21 oxen could eat up the whole pasture in 6 weeks, or 18 oxen in 9 weeks; what number of oxen, diminished by the removal of 9 at the end of 14 weeks, could eat it up in 18 weeks? *Ans.* 17 ox.

LXXXI. The present value of an annuity, to continue 23 years after the expiration of 8 years, is £55. 2*s*. 9½*d*. more than its value for 21 years after the expiration of 10 years; interest at 5 per cent. Required the annuity. *Ans*. £43. 16*s*. 3½*d*.

LXXXII. A person bought a quantity of brandy and rum for £10. 16*s*., and gave for the brandy 5*s*. 6*d*. and for the rum 2*s*. 10*d*. per bottle. He found moreover, that he could have bought as many bottles of rum as he now had of brandy, together with as many of brandy as he now had of rum, for £1. 12*s*. less. How much of each article was purchased?

Ans. B. 2½ doz.; R. 1½ doz.

LXXXIII. A, B, and C, working together, perform a certain work in a certain time. Find that time by the following data:—The work could have been performed in 14 days, if B and C had wrought 6 days before A joined them, or if A and C had wrought 7 days before B joined them, or if A and B had wrought 8 days before C joined them. *Ans*. 11$\frac{1}{13}$ days.

LXXXIV. A vessel, partly filled with water, is emptied by two pumps worked by M and N together, of whom M takes 5 strokes in the time of N's 4; but 20 of N's strokes discharge as much as 21 of M's. The vessel is emptied in 4 hrs. 40 min., and M throws out 84 gallons more than N. Required the hourly rate of discharge by each pump. *Ans*. M 112½, N 94½ gall.

LXXXV. By advices from Hamburgh, the price of *fine* gold there is 431½ marks banco per Cologne Mark Weight of 3608 Troy grains. Now, if in London the price of *standard* gold (containing 11 parts of pure gold and 1 of alloy) is 77*s*. 9*d*. per ounce, and if bills on London, payable at sight, sell at Hamburgh at 13 mks. 10½ sch. banco per £ sterling; whether is gold dearer in London or in Hamburgh, and how much per cent.? *Ans*. ·887 p. c. dearer in London.

LXXXVI. Each of two vessels contains 27 pints of

wine; and to one of these are added 13 pints of spirits. How much spirits must be added to the other, that the strength of the former mixture may be $\frac{1}{4}$ of that of the latter, supposing the strength of spirits to be 3 times that of wine? *Ans.* 453 pints.

LXXXVII. If the specific gravity of ivory is 1·825, of mahogany 1·063, and of cork ·24; how much cork must be added to 40 lbs. of ivory, that the united mass may weigh as much as an equal bulk of mahogany?
Ans. 4·87 lbs.

LXXXVIII. G has a yearly income of £500, receivable in equal payments at the end of every quarter. He defrays all his expenses at these quarterly periods; and the remaining sums being improved, at 5 p. c. per ann. simple interest, during the rest of the year, amount to £205. 12*s*. 6*d*. Required his total yearly expenditure, that of the first quarter being half as much as that of the second, the second half the fourth, and the third treble the first. *Ans.* £300.

LXXXIX. Suppose that in a meadow the grass is of uniform quality and growth, and that 6 oxen or 10 colts could eat up 3 acres of the pasture in $1\frac{1}{2}\frac{1}{7}$ of the time in which 10 oxen and 6 colts could eat up 8 acres; or that 600 sheep would require $2\frac{1}{7}$ weeks longer than 660 sheep to eat up 9 acres. In what time could 1 ox, 1 colt, and 1 sheep, together, eat up 1 acre of the pasture, on the supposition that 588 sheep eat as much in a week as 6 oxen and 11 colts? *Ans.* $9\frac{1}{3}\frac{7}{7}$ wks.

APPENDIX I.

Notes on the Tables of Money, Weight, and Measure.

Money.—The correct weight of the silver penny, in the Saxon times, was the 240th of the Saxon Pound (about 1-16th less than the present Troy Pound); hence the 240th part of the Troy Pound is called a *Pennyweight*. The Pound weight of silver was coined into 20 shillings; and the name *Pound* has been retained to denote the value of 20 shillings, although the value of the shilling has been much diminished.

As the British nation has chosen to have gold for its standard of value, the copper coinage is not considered a legal tender beyond 12*d*., nor the silver above 40*s*., for one payment.

The standard gold coin of this country consists of 22 parts of pure gold with 2 parts of copper alloy; this forms the arbitrary *assay* pound of 24 *carats* each 4 grains; and hence standard gold is said to be 22 carats *fine*.

The standard silver coin contains 37 parts of pure silver with 3 parts of copper alloy. Hence a lb. Troy of standard silver contains 222 dwts. of pure silver and 18 dwts. of alloy, and standard silver is said to be 11 oz. 2 dwts. *fine*.

The mintage rate of standard gold in Britain is 77*s*. 10½*d*. per ounce, and of standard silver 5*s*. 6*d*. per ounce. The Avoirdupois lb. of copper is coined into 24 pence.

The initial letters of the Latin words *Libra, Solidus, Denarius,* and *Quadrans,* are symbols for the meanings of these terms, denoting *Pound, Shilling, Penny,* and *Farthing,* respectively; but 1, 2, and 3, farthings are now represented by fractional parts of a penny: thus, instead of 12*s*. 5*d*. 2*q*., we write 12*s*. 5½*d*.

The following coins are occasionally referred to, though now obsolete:—A Groat = 4*d*.; A Noble = 6*s*. 8*d*.; An Angel = 10*s*.; A Mark = 13*s*. 4*d*.; A Guinea = 21*s*.

Troy Weight.—*Troy* and *Avoirdupois* are the only Imperial weights recognised by Act of Parliament.

The term *Troy* Weight was derived, as some say, from *Troyes*, the name of a town in France; but others refer to *Troy-novant*, the monkish name of London, as a more likely origin.

A cubic inch of distilled water, of the temperature of 62° Fahrenheit, when the barometer is at 30°, is understood to weigh 252·458 grains, which is the parliamentary standard for determining the weight of the Troy grain.

Apothecary's Weight, which differs from Troy weight only in the subdivisions of the ounce, is allowed, but not enforced by Government. It is employed for medical prescriptions; the ounce Troy being divided into 8 drams, the dram into 3 scruples, and the scruple into 20 grains.

Avoirdupois Weight.—The name Avoirdupois is with great probability traced to the Norman, *avoirs*, goods and chattels, and *poids*, weight.

7000 Troy grains make 1 lb. Avoirdupois; and as the lb. Troy contains 5760 grains, the lb. Troy is to the lb. Avoirdupois as 5760 to 7000, or as 144 to 175. Also, 480 grains make 1 oz. Troy, and 437½ grains make 1 oz. Avoirdupois; hence, the Troy oz. is to the Avoirdupois oz. as 960 to 875, or as 192 to 175.

Lineal Measure.—12 lines = 1 inch; 6 feet = 1 fathom; 69⅓ English miles = 60 nautical or geographical miles; hence a knot or nautical mile = 2025⅓ yards; 3 nautical miles = 1 league.

In Gunter's chain, used for land measuring, 7·92 inches = 1 link; 4 poles or 66 feet = 1 chain of 100 links; 80 chains = 1 mile.

The length of a pendulum vibrating seconds, in the latitude of London, is 39·1393 inches. This is the standard of lineal measure.

Superficial Measure.—10 chains in length by 1 in breadth = 10 square chains = 100,000 square links = 1 acre.

Solid Measure.—A cubic foot of distilled water, at the temperature of 62°, weighs about 1000 ounces Avoirdupois = 62½ lbs.; and this forms the basis of a Table of Specific Gravities. Thus, a cubic foot of mahogany weighs 1063 ounces, and its specific gravity is 1·063, *i. e.* its weight is 1·063 as great as that of an equal bulk of water.

Measure of Capacity.—5 Avoirdupois ounces of water, at 62°, = 1 gill; 4 gills = 1 pint; 10 lbs. Avoirdupois of water = 1 gallon = 277·274 cubic inches.

TIME.—In the time of Julius Cæsar the year was understood to consist of 365 da. 6 hrs., or 365¼ da. Therefore, according to the Julian Calendar, every 4th year is made to consist of 366 days, and each of the three preceding or following has 365. The year of 366 days is called *Leap* year, and is known by its number being exactly divisible by 4; thus A.D. 1847 is the 3rd year following a leap year; 1848 is a leap year. The leap year used to be called *Bissextile*, from two Latin words signifying *twice the sixth*, because the 6th day before the *Calends* or commencement of March, in the Roman Calendar, was repeated to introduce the additional or *intercalary* day. We add the leap-year day at the close of February.

The time, however, between the sun's leaving one point in the Ecliptic and returning to that point is 365 da. 5 ho. 48 min. 50 sec., which is therefore the true solar year;* and as this is 11 min. 10 sec. less than the Julian Account, the error of the latter will amount to about 3 days in 400 years. In rectification of this, Pope Gregory XIII., in the year 1582, ordained that the 5th of October should be reckoned as the 15th, and that thereafter, in every 400 years, 3 leap-year days should be suppressed, by accounting any year which denotes an exact number of centuries to be leap-year, only when the number of the century should be exactly divisible by 4. By this means 1600 and 2000 are made leap-years; whereas 1700, 1800, and 1900, are made common years, although according to the Julian scheme they would be leap-years.

The Gregorian Calendar was adopted in England in 1752, when the error had amounted to 11 days; and then the day following the 2nd of September was reckoned as the 14th. The *New Style*, as the Gregorian scheme is called, is now 12 days before the *Old* or Julian *Style*.

ANGULAR MEASURE.—The circumference of every circle is supposed to be divided into 360 degrees (360°), each degree into 60 minutes (60′), and each minute into 60 seconds (60″).

The French divide the circle differently, making the *grade*, or degree, the 400th part of a circle, the minute the 100th part of a degree, and the second the 100th part of a minute.

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* The *Sidereal* year of the earth, or that time which the Sun takes in moving through the Ecliptic from any fixed star to the same star again, is 365 da. 6 ho. 9 min. 10 sec. This contains an excess of 20 min. 20 sec. above the *Tropical* year, occasioned by the Precession of the Equinoxes, a slow retrograde motion of the Equinoctial Points, amounting to about 50·1 seconds of a degree in a year.

360° : 50·1″ : : 365 da. 6 ho. 9 min. 10 sec. : 20 min. 20 sec.

APPENDIX II.

On the Signs of Operation.

The principal signs of operation, employed in Arithmetic and Algebra, are +, —, ∽, ×, ÷, ∶, =, √, and ( ).

+, the sign of Addition, appears to have been originally the copulative conjunction &, a contraction of the Latin word *et*, being probably a form resulting from rapidity in writing the original character.

—, the sign of subtraction, is probably the result of a rapid formation of the letter *s* placed horizontally between the minuend and subtrahend; thus, 5 ∽ 3, *i. e.* 5 *subductis* 3, or 5 *subtracting* 3, might come to be represented by 5 — 3, just as 5 shillings are now frequently represented by 5/. Algebraists employ the Italic *ſ* horizontally, to denote subtraction, when it is not presumed which of two quantities is the minuend; thus, *a* ∽ *c* denotes the difference between *a* and *c*.

The signs + and — were first introduced in publication by Stifel, in 1544.

×, the sign of Multiplication, is said to have been first employed by Oughtred, in his *Clavis Mathematica*, 1631. It had been used, however, much earlier, as a sign of reciprocal multiplication, in the reduction of fractions to a common denominator, &c.* Thus, $\frac{2}{7} \times \frac{1}{4}$ originally signified the conversion of the numerators into 14 and 20, the form of the symbol being a natural indication of the process.

÷, the sign of Division, was derived from the Hindu practice of writing a dividend over its divisor.

The retention of dividend and divisor, to express Ratio, is represented by withdrawing the line from the points; thus, 16 ∶ 8 signifies $\frac{16}{8}$. Similarly, a double sign of division, ∺, having the line withdrawn, denotes Proportion, or Equality of Ratios; thus, 4 ∶ 3 ∷ 9 ∶ $6\frac{3}{4}$ denotes that $4 \div 3$ equals $9 \div 6\frac{3}{4}$. This use of the points was introduced by Oughtred.

=, the sign of Equality, was devised by Robert Recorde, who, in his *Whetstone of Witte* (the first English work on Algebra,

* This use of the sign is called by Bp. Tonstall 'multiplicatio in speciem crucis divi Andreæ.' *De Arte Supputandi*. Paris, 1529.

1557), tells us that two equal and parallel lines appeared to him the most natural symbol of equality.

$\sqrt{}$, the sign of Evolution, is a form of the initial letter of the Latin word *radix*, signifying *root*.

( ), the sign of Collection, is called a *Vinculum* (the Latin word for a *bond* or *tie*), and denotes that the sign preceding or following it has reference to the collected result of the quantities enclosed; sometimes the vinculum is in the form of a horizontal line placed over the quantities to be collected. Thus, $\sqrt{(16+9)}$, or $\sqrt{16+9}$, is equal to 5; whereas $\sqrt{16}+9$ denotes 13.

*** In a *System of Arithmetic* by William Ritchie, of the Perth Academy, there is a "Theory to account for the Origin of the Arithmetical Signs," p. 191; but it is very unsatisfactory. For other remarks on the invention of the signs, see De Morgan's *Notices of Arithmetical Books*, p. 19, *Stifel*.

APPENDIX III.

LIST OF ARITHMETICAL TREATISES.

*** I had intended a short history of Arithmetical Treatises to form one of the Appendixes to this volume. The interesting work lately published by Professor De Morgan has superseded that design; but the following alphabetical list of Arithmetical Works, being those which are in my own possession, will be found in some parts supplementary to De Morgan's book, while, at the same time, it will serve to indicate the principal sources of my information on the subjects contained in the Text-Book.

An asterisk is prefixed to the names of those authors whom De Morgan has omitted to notice.

*ABRAM, John, A Complete Treatise on Practical Arithmetic, containing, besides the Common Rules, New Principles of Mental, Visual, and Expeditious Calculation. *Canterbury*, 1842.

ADDINGTON, Stephen, A New and Complete System of A. *London*, 1765.

ARNOLD, Rev. Charles, The Boy's A. Parts 1 and 2. *London*, 1844-5.

BARREME, N., L'Arithmétique du Sr. Barreme. Nouvelle Edition. *Paris*, 1770.

BIRKS, Anthony and John, Arithmetical Collections and Improvements. *London*, 1766.

BONNYCASTLE, John, An Introduction to A. 8vo. Edition. *London*, 1810.

*** This edition contains a Table of all Prime Numbers from 1 to 99991, which is surpassed, however, in extent by some foreign Tables. The *Cribrum Arithmeticum* of L. Chernac (*Deventer*, 1811) extends to 1020000. For an account of the *κοσκινον Ερατοσθενους*, or method of *sifting* the Primes, see a Paper, by the Rev. Sam. Horsley, in the Philosoph. Transactions, vol. 62, p. 328; also Malcolm's Arith. p. 328; and Kavanagh's Arith., p. 58.

*BROOKS, William, Merchant's A., containing Remarks on Education, with Arithmetical and other Tables, for the use of Masters and their Pupils, Grocers, Farmers, Corn-factors, &c., with Mental Arithmetic, &c. *Bristol*, 1832.

*BUTLER, William, Arithmetical Questions, intended to answer the double purpose of Arithmetical Instruction and Miscellaneous Information. 8th Edition. *London*, 1822.

CAREY, George G., A Complete System of Theoretical and Mercantile A. *London*, 1818.

*CHAMBERS, William and Robert, An Introduction to A. *Edinburgh*, 1843. Arith. Theoretical and Practical, by W. Marr. *Edinburgh*, 1847.

*CHIESMAN, John, The Arithmetical Repository, or Student's Guide to Commerce and Science. *London*, 1833.

CLARE, Martin, Youth's Introduction to Trade and Business. 12th Edition, with Additions, including Solutions to the most difficult Questions, by Benjamin Webb. *London*, 1791.

*** The original edition of Clare's Introduction was in 1739. Jacob Welsh, in the Preface to his Arithmetic (1758), says, "Mr. Clare's Arithmetic undoubtedly contains a large collection of the most ingenious Questions solvable by common Arithmetic, yet published in our language." Vyse, in his Tutor's Guide, has adopted all Clare's Questions.

*COLENSO, Rev. J. W., Arithmetic designed for the use of Schools. *London*, 1843.

CONDILLAC, M. L'Abbé de, La Langue des Calculs. *Paris*, An. vi. de la République (1797).

*COTTERILL, Rev. J., The Rule of Three, not the Rule of Proportion, but a Rule illustrating Proportion, and a Method of Solution proposed which does not require the use of Proportion. *London*, 1844.

*CRAIGHEAD'S Arith., edited by James Nicolson, Bookseller, Dundee. *Edinburgh*, 1798.

*Crossley, John Thomas, and William Martin, The Intellectual Calculator, or Manual of Practical A. 17th Edition. *London*, n. d.

Cunningham, Robert, The Arithmetical Text-Book. *Edinburgh*, 1832.

*Darnell, George, Arith. made intelligible to Children. Parts 1 and 2. *London*, 1847.

*Davidson, John, The Practical Calculator, or a Course of Arithmetic, Algebra, and Mensuration. *Edinburgh*, 1809. Supplement to the Pract. Calcul., containing Answers and Solutions. *Edinburgh*, 1814.

*** This excellent work was afterwards remodelled, and resolved into two separate Treatises, 'Practical Mathematics,' and 'Arithmetic Modernized,' both of which are, at the present time, very popular school-books.

*Davidson, John, and Robert Scott, Arithmetic Modernised, or a complete system of A., adapted to Modern Practice. 8th Edition. *Edinburgh*, 1844.

*Davidson, John, The Young Arithmetician's Guide. 2nd Edition. *Edinburgh*, 1834.

Davidson, John, The Young Mental Calculator's Guide, being a Course of Mental A. for the use of Schools. *Edinburgh*, 1836.

De Morgan, Augustus, The Elements of A. 5th Edition. *London*, 1846.

*** Admirably adapted as a discipline.

Dubost, Christopher, Commercial A. 2nd Edition. *London*, 1807.

*Goymer, E. N., The British Youth's Practical Guide to A. *Ipswich*, 1832.

Gray, James, An Introduction to A. 42nd Edition. *Edinburgh*, 1838.

*Grémilliet, J. J., Recueil de Problèmes amusans et instructifs, avec les Démonstrations Raisonnées, et l'Application des Règles de l'Arithmétique à leurs Solutions. 1re Partie, contenant les Questions; 2me Partie, contenant les Solutions. 4me Edition. *Paris*, 1828.

*** This work contains 1320 Problems with their solutions, and is an excellent collection; but the author occasionally discourages, in an improper degree, the employment of Algebra, and gets involved in rather tedious arithmetical demonstrations. However superior, in respect to intellectual exercise, arithmetical solution may be, there is a province assigned to alge-

bra, which arithmetic, properly so called, cannot legitimately nor successfully invade.

*Guyot, L. V., Traité Général et Elémentaire d'Arithmétique. *Paris*, 1825.

Halbert, William, The Practical Figurer, or an Improved System of A. *Paisley*, 1789.

*** The author says, "When a boy has been accustomed to resolve puzzling and amusing questions, such as are connected with the real occurrences of life become entirely easy to him." The book accordingly contains many entertaining problems. In the list of Subscribers to the work, I observe the illustrious name *Robert Burns of Parnassus*.

Hamilton, Dr. Robert, An Introduction to Merchandise, containing the Theory and Practice of Arith., Algebra, Commerce, &c. New modelled by Elias Johnston. *Edinburgh*, 1820.

Hamilton's Arithmetic, enlarged and improved by the Rev. James Welsh. 2nd Edition. *Aberdeen*, 1832.

Harris, James, Questions in A., comprising examples in all the Rules of the Science. *Cambridge*, 1841.

*** A useful collection of elementary problems in Arithmetic, Mechanics, and Hydrostatics.

Harrison, Daniel, A New System of Mental A. *London*, 1837.

*** Its chief feature is a novel method of Reduction called the *Quadrantal System*; but, although the work contains some very eligible modes of abbreviating calculation, I cannot see why Davies should have considered it "the very best work on Arithmetic for commercial purposes." *See* Hutton's Course, by Davies, vol. i. p. 53. De Morgan justly observes, in reference to Harrison's work, that books of mental arithmetic choose their own examples, and thus make their own rules work well. *Arithmetical Books*, p. 94.

*Haxton, A., The Elements of A. *Edinburgh*, 1827.

Hay, Richard, The Beauties of A., being Solutions of above 600 of the most difficult unsolved Questions selected from the most eminent Authors. *Edinburgh*, 1816.

*Hiley, Richard, The Arithmetical Companion, intended, by Miscellaneous Examples, to perfect the Pupil in a knowledge of Arith. 2nd Edition. *London*, 1846.

Hill, John, Arithmetic both in the Theory and Practice. 10th Edition. *Edinburgh*, 1760.

*Hind, John, The Principles and Practice of A. 4th Edition. *Cambridge*, 1842.

Hutton, George, A Manual of A. *London*, 1844.

*Ingram, Alexander, The Principles of A. 7th Edition. *Edinburgh*, 1831.

*Irish National Schools' Treatise on A. 3rd Edition. *Dublin*, 1845.

*Johnston, Elias, The Elements of A. *Edinburgh*, 1826.

*Johnstone, R., A Treatise on Mental A. *Edinburgh*, 1835.

*Kavanagh, James W., Arith., its Principles and Practice. *Dublin*, 1844.

Keith, Thomas, The Complete Practical Arithmetician. 7th Edition. *London*, 1819.

King, John, An Essay or Attempt towards establishing a New Universal System of A.; Division of the Year, Circle, and Hour, &c.; and introducing some necessary alterations tending to simplify the present Scale of Music; Remarks on the new Calendar of the French Republic; &c. 2nd Edition. *Buckingham*, 1808.

Kirkby, Rev. John, Arithmetical Institutions. *London*, 1735.

Lacroix, S. F., Traité Elémentaire d'Arithmétique. 8me Edition. *Paris*, 1808. An Elementary Treatise on the Mathematical Principles of A., translated from the 15th French Edition. *London*, 1823.

Lardner, Rev. Dionysius, A Treatise on A. Theoretical and Practical. *London*, 1834.

*Macgowan, Rev. James, First Lessons in A., on the plan of Perfect Progressive Instruction. *London*, 1829.

*Macleod, Walter, Arithmetical Questions, comprising a Systematic Course of Mental Arithmetic. Part 1. Whole Numbers. 3rd Edition. *London*, 1846. Part 2. The Theory and Practice of Fractional Arithmetic. *London*, 1845.

*** These questions are of an elementary character, progressively arranged, and adapted to the method of Pestalozzi. As furnishing a means of intellectual discipline for children, they are well worthy of adoption in schools.

*Macmillan, Alexander, Arithm. with its General Application, and its Application to Trade. *Dumfries*, 1811.

Malcolm, Alexander, A New System of A. Theoretical and Practical, wherein the Science of Numbers is demonstrated in a Regular Course from its First Principles,

through all the Parts and Branches thereof. *London*, 1730.

*** One of the most complete and rigid expositions of Theoretical Arithmetic which has ever been published.

*MARR, *see* CHAMBERS.

*MARTIN, John, The Seaman's A., or the Application of the several Rules of that Science to the Purposes of the Maritime Profession. *Edinburgh*, 1843.

MELROSE, A., A Concise System of Practical A. Edited by Alexander Ingram and James Trotter. 19th Edition. *Edinburgh*, 1841.

MOLE, John, Elements of Algebra; to which is prefixed a choice Collection of Arithmetical Questions with their Solutions, including some new improvements worthy the attention of Arithmeticians. *London*, 1788.

*MORRISON, James, The Accomptant's Guide, an improved System of Practical A. 2nd Edition. *Glasgow*, 1806. Key to Ditto. 1807.

MORRISON, James, A Concise System of Commercial A., adapted to Modern Practice. New Edition. *London*, 1842.

*MORRISON, C., The Elements of Practical A. simplified, intended as an Introduction to the Counting-house. *Edinburgh*, 1830.

MOSLEY, Rev. Richard, Elements of A. *London*, 1836.

*MUDIE, Robert, The Winchester A. of Numbers and Quantities, on Improved Principles. *Winchester*, 1841.

*MURRAY, Commercial A. *Edinburgh, circa* 1830.

*NESBIT, Anthony, A Treatise on Practical A. Part I. 3rd Edition. *London*, 1839. Part 2. *London*, 1846.

*** One of the best Commercial Treatises of the present day.

NEWTON, Isaac, Universal A. Raphson's Translation, revised by Samuel Cunn. *London*, 1720.

*** Chiefly on Algebra and Geometry.

*ORAM, Rev. H. A., Examples in A. systematically arranged in Four Parts. For the use of Proprietary Schools. *London*, 1844.

PORTER, Mrs. G. R., Conversations on A. *London*, 1835.

RECORDE, M. Robert, The Grounde of Artes, teachinge the Worke and Practise of Arithmetike. Edited by I. D. (Dr. John Dee). *Londini*, 30 Nov. 1570.

*** The original edition, as Dee states, was in 1540, but appears to be now unknown. Buckley, Digges, and Recorde are commonly referred to as the earliest English writers on Arithmetic.

REYNOLDS, S. P., A Treatise on Practical A. and Mensuration. 7th Edition. *London*, 1842.

*RICHSON, Charles, Mental A. and Expeditious Calculation, adapted to Real Business. *London*, 1835.

RITCHIE, William, A System of A. Theor. and Pract. *Edinburgh*, 1818.

*SAIGEY, M., Problèmes d'Arithmétique et Exercises de Calcul sur les Questions Ordinaires de la Vie; sur la Géomètrie, la Mécanique, l'Astronomie, la Géographie, la Physique, &c. 4me Edition. *Paris*, 1841. Solutions Raisonnées des Problèmes d'Arithm. de M. Saigey, par M. SONNET. 2de Edition. *Paris*, 1841.

SAUL, Joseph, The Tutor and Scholar's Assistant, being a Complete Treatise on Vulgar and Decimal A. 12th Edition. *London*, 1839.

*SCOTT, William, Elements of A. and Algebra. For the use of the Royal Military College, Sandhurst. *London*, 1844.

*** A very elaborate and voluminous exposition of Principles.

*SHORT, Dr. Thomas Vowler, Hints on Teaching Vulgar and Decimal Fractions. *London*, 1840.

*** A skilful Treatise, of great practical excellence.

*TATE, Thomas, Exercises in A. for Elementary Schools; after the Method of Pestalozzi. Under the Sanction of the Committee of Council on Education. *London*, 1844.

*** The most complete and systematic exposition which Pestalozzian A. has yet received. As a discipline for the young it is of great value.

TATE, Thomas, A Treatise on the First Principles of A., after the Method of Pestalozzi. 2nd Edit. *London*, 1847.

TATE, William, A System of Commercial A. *London*, 1810.

*TATE, William, The Counting-house Guide to the Higher Branches of Calculations. Part 1st, forming an Appendix to the Elements of Comm. Arithm. Part 2nd, forming a Supplement and Key to the Appendix. *London*, 1844.

*** Works of great commercial utility.

TAYLOR, Thomas (the Platonist), Theoretic A. in Three Books; containing the substance of all that has been written on this subject by Theo of Smyrna, Nicomachus, Iamblicus, and Boetius, &c. *London*, 1816.

*** The curious production of a singularly constituted mind, but of no practical utility as an Arithmetical work. Horsley says, "Nichomachus Gerasinus was a shallow writer of the 3rd or 4th century;

and Boethius's Treatise on Numbers is but an abridgment of the wretched performance of Nichomachus."—Phil. Trans., vol. lxii.

THOMSON, Dr. James, A Treatise on A. in Theory and Practice. 23rd Edition. *London*, 1842.

*** A treatise of well-known excellence.

*THOMSON, R., The Arithmetical Repository, being Solutions of the most difficult unsolved Questions selected from various Authors. *Edinburgh*, 1811.

TINWELL, William, A Treatise on Practical A. 6th Edition. With improvements by And. Tinwell. *Newcastle*, 1811.

TONSTALL, Cuthbert, De Arte Supputandi Libri Quatuor. *Parisiis* excudebat *Robertus Stephanus*, ann. 1529, Prid. Id. Jun.

*** Tonstall's book was first published in London in 1522, and the Paris edition appears to be just a reproduction of the original work. It is a truly classical performance, and will be considered a good book of Arithm., even when compared with our best modern Treatises, although it is the first that was published on the subject in England. Tonstall was Bishop of Durham (successor to Cardinal Wolsey) when he composed this work, and he committed it to the press as the fruits of a species of study which he meant thenceforth to abandon, being appointed to the vacant see of London, and considering, as he says in his Dedication to Thomas More, that the remainder of his life should be devoted to sacred literature.

VYSE, Charles, The Tutor's Guide. 14th Edition. *London*, 1810.

*WAYMOUTH, John Davis, Arithmetical Exercises, consisting of 430 Practical Questions. *London*, 1844.

WELSH, Jacob, The Schoolmaster's General Assistant, or a Complete System of A., in two volumes. *London*, 1758.

*** The 1st volume contains some amusing original problems; the 2nd, which is occupied with short methods of calculation, is of very little worth. It appears, however, that this author was the first to suggest the useful species of abbreviated multiplication which we have exemplified in p. 22 of the Text-Book.

WHITE, Rev. John, The Tutor's Expeditious Assistant; being a System of Practical A., founded on a New Discovery, and arranged so as to enable the Schoolmaster, at the mere

glance of any sum therein, to ascertain its correctness, &c. 3rd Edition. *London*, 1841.

*** The 'New Discovery' is a species of Artificial Test, which was also employed by Mr. Miller, of Dundee, in a set of Arithmetical sheets, published many years since. It is of very limited utility.

WILLIAMS, Rev. David, A New System of Practical and Scientific A., with an Appendix of 400 Miscellaneous Questions. *London*, 1818.

WILSON, John, an Introduction to A. 2nd Edition. *Edinburgh*, 1752.

*WISEMAN, R., The Arithmetician's Text-Book. *St. Andrew's*, 1797.

*WRIGHT, J. M. F., Self-Instructions in Pure A., designed for Schools and Universities. *Cambridge*, 1829.

APPENDIX IV.

SOLUTIONS OF THE MORE DIFFICULT EXERCISES IN THE PUPIL'S MANUAL.

*** For remarks on the construction of some of the Exercises in Simple Addition, Multiplication, and Division, the Teacher is referred to the close of the volume.

PROMISCUOUS EXERCISES, p. 27.

Ex. 19. 35 qui. at 8*d.* = 280*d.*, which, at 10*d.* a quire, would buy 280 ÷ 10, or 28 quires.

20. 1400 land mi., each 1760 yds., = 2464000 yds., which is reduced, by division, to knots of 2000 yds. each.

21. In 14 days the gain is 1 min. 38 sec., or 98 sec. = 7 sec. per day.

22. 9 bu. at 42*d.* = 378*d.* worth of cherries at 8*d.* a-lb.; hence 378 ÷ 8, the quantity in lbs.

23. Saves 3*s.* 6*d.* a fortnight, or 1*s.* 9*d.* a week, which in 50 guin. is contained 600 times.

24. 10 chairs at 19*s.* 6*d.* + 2 at 27*s.* = £12. 9*s.*, to be deducted from £16. 5*s.* 6*d.*, leaving £3. 16*s.* 6*d.*

25. 9909; + 12000 + 1212; + 15000 + 1515.

26. £11. 8*s*. − £9. 19*s*. 6*d*. = £1. 8*s*. 6*d*.

27. 5 at 4½*d*. + 11 at 3*s*. 6*d*. = 40*s*. 4½*d*., which divide by 4*s*. 9*d*.

28. £106. 4*s*. ÷ 32 is £3. 6*s*. 4½*d*. per piece, which divide by 2*s*. 5½*d*.

29. 450 ft. of wood offered for 126 gall.; 450 ft. at 2*s*. 11*d*. is £65. 12*s*. 6*d*., the 126th of which is the price per gall.

30. Add 32*s*. 6*d*. × 26 to 43*s*. × 23, and divide the sum by 14*s*.

31. The size of the farm + 42 acres = 9 times its size; therefore 42 acres must be 8 times its size; 42 ac. ÷ 8.

32. The difference of the ages is 32; and as the father is 3 times as old as the daughter, the difference of the ages is twice the age of the daughter; 32 ÷ 2 = 16, the daughter's age.

33. The remainder is 80, the product 80 + 7, and the original number is 87 ÷ 3.

34. The sum is 60, the quotient 60 − 46, and the original number is 14 × 8.

35. The 5th of 4680 is 936, which divided by 3, the quotient diminished by 36, produces 276.

36. Retracing the process, 47 × 6 − 17 = 265, which divided by 5 gives 53.

37. The two dig 62 yds. per day, or they dig 372 yds. in 372 ÷ 62 days.

38. From 1st Jan. to 1st Aug. are 212 days ÷ 7 = 30 weeks + 2 days; and 2 days beyond an exact number of weeks is 2 days beyond Wednesday, viz. Friday = 1st August.

39. From 1st Jan. to 18th June are 168 days ÷ 7 = 24 weeks, an exact number; therefore the 18th of June falls on the same day of the week as that which begins the year.

40. From 1st Jan. to 28th Sept., leap year, are 271 days ÷ 7 = 38 weeks + 5 days; and the 5th day before Saturday is Monday.

41. Of 78 glasses he earns 8 at 7¼*d*. + 65 at 14½*d*. = £4. 3*s*. 6½*d*., from which deduct 5 at 4*d*. = 1*s*. 8*d*.

42. £1. 1*s*. taken the required no. of times + 5*s*. taken the same no. of times + 1*d*. taken the same no. of times = £1. 6*s*. 1*d*. taken that no. of times = £97. 16*s*. 3*d*.; therefore 23475*d*. ÷ 313*d*. = 75.

43. 1 wk. + 1 da. + 1 ho. taken the required no. of times, amounts to 386 days; therefore 386 × 24 ho. ÷ 193 ho. = 48.

44. 7 da. wages at 5*s*. + 2 da. loss of income at 4*s*. 6*d*. = 44*s*., which is 6*s*. 6*d*. more expensive than 5 da. wages at 7*s*. 6*d*.

Simple Proportion.

Ex. 36. £5 : £8. 2*s.* 6*d.* :: £100; or 5 : 100 :: £8. 2*s.* 6*d.* : 20 times 162*s.* 6*d.* = £162. 10*s.*

37. 20*s.* : £641. 18*s.* 4*d.* :: 12*s.* 6*d.*; or 8 hf.-crs. : 5 hf.-crs. :: £641. 18*s.* 4*d.*

38. Calculate, first, 41 E. e. 4 qrs. = 209 qrs. at 19*s.* 9½*d.* per ell, or 3*s.* 11½*d.* per qr., and then divide by 17*s.* 5*d.*, *i. e.* (47½*d.* × 209) ÷ 209*d.* = 47½ yds.

Or, 17*s.* 5*d.* per yd. : 3*s.* 11½*d.* per qr. :: 209 qrs. : 47½ yds.

39. 2555*d.* : 2765*d.* :: 73 boys : 79 boys;
511 553
73 79 ÷ 5 = 15 men, 4 boys.

40. £219 : £840 :: 146 da. : 280 × 2 da.
73 280

41. £72. 10*s.* 7½*d.* : £52. 15*s.* :: 165 yds.
2321 1688
211 8 × 15 = 120 yds.

42. £3. 14*s.* 6*d.* − £2. 10*s.* = 24*s.* 6*d.* gain per cwt.
24*s.* 6*d.* : 42*s.* :: 1 cwt.
49 84 ÷ 49 = 12 ÷ 7 = 1 cwt. 2 qrs. 24 lbs.

43. 29*s.* 6*d.* : £83. 6*s.* 9*d.* :: 1 doz. : 56½ doz. sold.
118 6667

44. 12 men could do the remaining work in the remaining 18 days, but it is assigned to 7 men; therefore,
7 m. : 12 m. :: 18 da. : 30⅞ da.

45. 15 men could do the remaining work in the remaining 10 days, but it is to be finished in 6 days; therefore,
6 da. : 10 da. :: 15 m. : 25 m., or 10 added.
otherwise; 6 da. : 4 da. :: 15 m. : 10 m.
i. e. the work that would have been done by 15 men in 10 − 6 days, is distributed over the 6 days by the additional work of 10 men.

46. 70 da. : 60 da. :: £182 : £156 lent by T for 70 days would have cancelled the original obligation; but T lent only £65 for that period; therefore, he must now lend £35 for a time equivalent to £156 − 65 for 70 days;
£35 : £91 :: 70 da. : 182 da.

47. A travels 23 × 4 = 92 mi. before B's outset, and this is to be gained by B at the rate of 9 miles gained per day, *i. e.* 32 − 23; therefore, 9 mi. : 92 mi. :: 1 da.

48. The supplying pipe runs in 140 min. as much as the discharging pipe runs in 300 min.; therefore, the former would run 30 gallons in the time that the latter would run 14 gallons; that

is, of every 30 gallons run by the supplying pipe 16 gallons remain in the cistern; accordingly,

30 : 16 :: 60 gall. : 32 gall.

49. The additional 63 men could do in 40 days what the required number of men can do in the additional 12 days; hence,

12 da. : 40 da. :: 63 m. : 210 m.

50. First, 1200 men work for 2 weeks.

Secondly, The remaining work would be done by 1200 men in 10 weeks; or 1500 men would do it in 8 weeks, of which time they work 2 weeks.

Thirdly, The work still remaining would be done by 1500 men in 6 weeks; or 1000 men do it in 9 weeks.

Therefore, the whole work is done in 2 weeks of 1200 men + 2 weeks of 1500 men + 9 weeks of 1000 men.

Compound Proportion.

Ex. 7. $\left.\begin{matrix} 6d. : 7d. \\ 29s. : 37s. \end{matrix}\right\}$ :: 35 ox. : *Ans.*

8. $\left.\begin{matrix} 93 \text{ mi.} : 124 \text{ mi.} \\ 1552 \text{ hf. d.} : 1455 \text{ hf. d.} \end{matrix}\right\}$ :: 26 qrs. : *Ans.*

Cancel 3 in 93 and 1455; 4 in 124 and 1552; and 97 in 388 and 485.

9. $\left.\begin{matrix} 48 \text{ m.} : 32 \text{ m.} \\ 40 \text{ da.} : 45 \text{ da.} \end{matrix}\right\}$ :: 2 lbs. : *Ans.*

Cancel 4 in 48 and 32; and 5 in 40 and 45.

10. $\left.\begin{matrix} 27 \text{ pa.} : 52 \text{ pa.} \\ 8 \text{ lb.} : 10 \text{ lb.} \\ 8 \text{ lb.} : 9 \text{ lb.} \end{matrix}\right\}$ :: £51 : *Ans.*

Cancel 3 in 27 and 51; and 8 in 8 and 52 × 10.

11. $\left.\begin{matrix} 15 \text{ m.} : 6 \text{ m.} \\ 4 \text{ ti.} : 3 \text{ ti.} \\ \text{G. } 18 \times 21 : £24 \times 20 \\ 8 \text{ ho.} : 9 \text{ ho.} \end{matrix}\right\}$ :: 21 da. : *Ans.*

Cancel 5 in 15 and 20; and 3 in 18 and 24.

12. $\left.\begin{matrix} 5 \text{ pks.} : 3 \text{ pks.} \\ 20 \text{ gal.} : 18 \text{ gal.} \\ 78s. : 65s. \end{matrix}\right\}$ :: 10*d.* : *Ans.*

Cancel 2 in 20 and 18; and 13 in 78 and 65.

13. $\left.\begin{matrix} 5 \text{ ox.} : 14 \text{ ox.} \\ 24 \text{ ho.} : 25 \text{ ho.} \\ 70\ldots\text{grs.} : 76\ldots\text{grs.} \end{matrix}\right\}$ :: 156 lbs. D. : *Ans.*

Cancel 14 in 14 and 70; and 12 in 24 and 156.

14. £16 : £6 } : : £600 : A's loan.
5 mo. : 8 mo. }

Cancel 2 in 16 and 6; and 5 in 5 and 600.

£16 : £7 } : : £600 : B's loan.
9 mo. : 8 mo. }

Cancel 2 in 16 and 600; and 3 in 9 and 300.

15. If 2 adults = 3 children, then 6 adults = 9 children; and 6 adults + 5 children = 14 children.

98*d.* : 153*d.* }
15 da. : 7 da. } : : 14 children : 9 children;
17 hf. d. : 15 hf. d. } 9 − 3 = 6 ch. = 4 adults.

16. If 15*s.* gain 6*d.* in 6 mths., in what time should 40*s.* gain 2*s.*?

40*s.* : 15*s.* } : : 6 mo. : *Ans.*
6*d.* : 24*d.* }

17. Say, If 4 sticks of wax = 9 penc., then fewer = 7 penc.; if fewer = 17 quills, then more = 40 quills; if more = 3 quires, then still more = 5 quires. These considerations give the following ratios:—

9 pen. : 7 pen. }
17 qls. : 40 qls. } : : 4 st. : *Ans.*
3 qui. : 5 qui. }

18. 1 cu. : 8 cu. }
5 al. : 7 al. } : : 10*d.* : *Ans.*
16 ra. : 15 ra. }
14 su. : 6 su. }

19. B 6 : B 4 }
C 3 : C 8 } : : A 5 mi. : *Ans.*
D 10 : D 18 }

Promiscuous Exercises in Vulgar Fractions.

Ex. 3. Spent $\frac{19}{36}$; therefore 1 − $\frac{19}{36}$ remains.

4. 9 : 30 : : 5 : *Ans.*

5. $\frac{1}{8}$ of a yd. costs 5$\frac{3}{4}$*d.*, which deduct from 2*s.* 10$\frac{1}{2}$*d.*

6. Add 33$\frac{1}{8}$ to 4 times 27$\frac{3}{16}$.

7. 21 ÷ 52$\frac{1}{2}$ = 2 ÷ 5 = $\frac{1}{2}$ of *twice*, or $\frac{2}{5}$ of *once*.

8. 41 cwt. − 40 cwt. = 1 cwt. = 40 *fortieths* of 1 cwt.

9. Reduction gives 3 ro. 30 po. 26$\frac{1}{2}$ yds. 3 ft. 34 in.
or, $\frac{1}{2}$ sq. yd. = 4 72

3 ro. 30 po. 26 yds. 7 ft. 106 in.

10. 41 *sixty-thirds* : 63 *sixty-thirds* :: 4*s.* $3\frac{1}{2}$*d.* : *Ans.*

11. $\frac{7}{20} \times \frac{16}{17}$. 12. 54...240ths $= 2\frac{7}{10}$ *twelfths.*

13. $\frac{1}{4}$ mi. $= \frac{2}{7}$ fur.; 7 : 8 :: 8 : $9\frac{1}{7}$ *eighths.*

14. 4840 sq. yds. = 1 ac.; (4840 × 27) *twenty-sevenths* of 1 sq. yd. = 1 ac.; $\frac{2}{7}$ of (4840 × 27) = *Ans.*

15. $\frac{3\frac{3}{4}}{12} \div \frac{5}{8} = \frac{15}{50} \times \frac{8}{5}$.

16. £32 — £26 = £6 = (6 × 25) *twenty-fifths* of £1.

17. Whole sum $= 1 + \frac{3}{10} + 2\frac{1}{2}$ times $\frac{3}{10} = 2\frac{1}{20}$ times A's share.

18. $\frac{3}{20}$*d.* = £$\frac{1}{2000}$; $\frac{3}{20}$*s.* = £$\frac{3}{200}$ or £$\frac{12}{2000}$; subtract.

19. $\frac{3}{5}$ of $\frac{7}{9}$, or $\frac{7}{15}$ of the farm = 140 ac.; therefore 1 ninth = 20 ac.; and 9 ninths = 180 ac.

20. $1\frac{7}{11}$*s.* : $2\frac{1}{2}$*s.* :: $\frac{3}{5}$ lb. : $\frac{3}{5} \times \frac{25}{9} \times \frac{11}{18}$ lb.

21. $15\frac{3}{4}$ grs. : $\frac{3}{4} \times 24$ grs. :: $2\frac{1}{6}$*d.* : $\frac{2}{6} \times \frac{3}{4} \times 24 \times \frac{3}{47}$*d.*

22. 351*d.* : 153*d.* :: $10\frac{7}{17}$ yd. : $\frac{177 \times 9}{351}$.

23. $\frac{1}{12}$ qr. × 32 = $\frac{3}{4}$ pk.; $\frac{3}{4}$ bu. × 4 = $\frac{6}{4}$ pk.; subtract.

24. Diff. of 1 yd. and 1 ft. = 24 in.; $\frac{3}{5}$ of 24 in. = *Ans.*

25. $\frac{1}{2} + \frac{3}{5} + \frac{3}{4} = \frac{41}{20} = \frac{1}{20}$ more than the whole.

26. $\frac{1}{2}$ of $\frac{3}{5}$, or $\frac{1}{5}$ of the money = $3\frac{1}{2}$*d.*; $3\frac{1}{2}$*d.* × 6 = *Ans.*

27. $\frac{3}{4}$ of $\frac{3}{5}$, or $\frac{1}{5}$ of the journ., leaves $\frac{7}{8}$ of it = $10\frac{1}{2}$ mi.
7 : 8 :: $10\frac{1}{2}$ mi. : 12 mi.

28. $\frac{3}{7} - \frac{3}{28}$, or $\frac{1}{28}$ of the No. = 20; $\frac{1}{28}$ of it = 4; &c.

29. $\frac{3}{5} + \frac{3}{5}$ of $\frac{3}{4} = \frac{18}{20}$, leaving $\frac{2}{20}$ of the money = 15*s.* $4\frac{1}{2}$*d.*; therefore 9 : 25 :: 15*s.* $4\frac{1}{2}$*d.* : *Ans.*

30. The sums taken away are £22. 11*s.* 6*d.* and £10. 15*s.*;

£37.	12*s.*	6*d.*	:	£22.	11*s.*	6*d.*	::	1	:	$\frac{3}{5}$;
37	12	6	:	10	15	0	::	1	:	$\frac{3}{5}$.

31. $\frac{1}{2} - \frac{1}{4}$ of $\frac{3}{5}$, or $\frac{1}{20}$ of the money = 12*s.* 6*d.*; $\frac{1}{21} = 2s.\ 6d.$; &c.

32. B will arrive in $5 \div 3\frac{1}{3}$ or $1\frac{1}{2}$ hour; A must travel $6\frac{3}{4}$ mi. in $1\frac{1}{2}$ hour; therefore, $\frac{27}{2} \div \frac{3}{2} = Ans.$

33. $1 + \frac{1}{3}$ of my age = $13\frac{1}{3}$; $13\frac{1}{3} \div 1\frac{1}{3} = 11\frac{1}{4}$.

34. $1 - \frac{1}{4}$ of my age = 14; $14 \div \frac{3}{4} = 17\frac{1}{2}$.

35. $\frac{1}{7} + \frac{2}{7}$ of $\frac{9}{11}$, or $\frac{27}{77}$ of the money = 3*s.* $7\frac{1}{2}$*d.*
29 : 14 :: 3*s.* $7\frac{1}{2}$*d.* : 1*s.* 9*d.*

36. The second horse clears $7\frac{1}{2}$ mi. in 28 min. = $8\frac{4}{7}$ mi. in 32 min.; therefore the speed of the first is to that of the second as 9 : $8\frac{4}{7}$, or as 21 : 20.

37. $\frac{1}{3}$ of $\frac{3}{7}$ of the quantity = $\frac{1}{4}$ cwt.; or $\frac{3}{7}$ of the quant. = 1 cwt.; or 3 times the quant. = 7 cwt.

38. The given sums, reduced to 37ths of a penny, are 13024 and 9240; therefore,

$$\left.\begin{matrix} 9240 : 13024 \\ 5\tfrac{1}{3}\text{ oz.} : 60\text{ oz.} \end{matrix}\right\} :: 3\tfrac{1}{2}\text{ oz.} : \frac{7 \times 6512 \times 60}{3080 \times 16}\text{ oz.}$$

39. $\left.\begin{matrix} 5\text{ men} : 7\text{ men} \\ 13\text{ parts} : 15\text{ parts} \\ 4\tfrac{1}{2}\text{ da.} : 3\tfrac{6}{11}\text{ da.} \end{matrix}\right\} :: 8\tfrac{1}{2}\text{ho.} : \frac{33 \times 7 \times 15 \times 39 \times 9}{4 \times 11 \times 5 \times 13 \times 40}\text{ho.}$

40. $\frac{7}{9}$ oz. : $35\frac{3}{15}$ oz. :: $3\frac{3}{4}d.$: $\frac{15 \times 527 \times 9}{4 \times 15 \times 7}\,d.$

$\frac{18}{17}$ oz. : $54\frac{2}{3}$ oz. :: $3\frac{3}{4}d.$: $\frac{15 \times 272 \times 7}{4 \times 5 \times 48}\,d.$

$(4743 + 833)\,d. \div 28 = 199\frac{1}{4}d.$

41. The rates per day are $\frac{1}{56}$ of the work by A, and $\frac{1}{42}$ by B; therefore, both together do $\frac{13}{252}$ of the work per day;

13 parts : 252 parts :: 1 da. : *Ans.*

Otherwise:—Since 36 da. of A = 42 da. of B, therefore, 36 da. of A + 42 da. of A = 42 da. of B + 42 da. of A; or 78 da. of A = 42 da. of A + B; accordingly,

78 da. of A : 36 da. of A :: 42 da. of A + B : *Ans.*

42. The horses can eat $\frac{1}{72}$, and the oxen $\frac{1}{84}$, of the hay, per hour; therefore, both together can eat $\frac{11}{168}$ of it per hour;

11 parts : 168 parts :: 1 ho. : *Ans.*

Otherwise, as in last Question:—

(12 + 54) ho. of horses : 12 ho. of do. :: 54 ho. of both : *Ans.*

43. The first party could reap $\frac{1}{42}$, and the second party $\frac{1}{84}$, of the crop, per day; or both together $\frac{1}{105}$ of the crop per day; but the three parties together do $\frac{1}{14}$ per day; therefore, the third party could reap $\frac{1}{14} - \frac{1}{105} = \frac{1}{30}$ per day;

$\frac{1}{30}$ crop. : $\frac{1}{2}$ crop :: 1 da. : 15 da.

Otherwise:—Calling the parties A, B, and C, we may find, as in Qu. 41, that A and B together could reap the crop in $26\frac{1}{4}$ da.; for, 45 + 63 : 45 :: 63 da. : $26\frac{1}{4}$ da.

Now, as A + B + C take 14 da., and A + B would take $26\frac{1}{4}$ da., it is evident that C can do $12\frac{1}{4}$ days' work of A + B in 14 days; therefore,

$(26\frac{1}{4} - 14)$ da. of A + B : $26\frac{1}{4}$ da. of do. :: 14 da. of C : 30 da. of C, to cut down *all* the crop.

44. In 3 yrs. I save $3 - 2\frac{1}{2}$, or $\frac{1}{2}$ of my annual income; therefore, in 6 months I save $\frac{1}{12}$ of my annual income, or $\frac{1}{9}$ of my monthly income, or $\frac{9}{11}$ of 9 months' income.

45. $\left.\begin{array}{lll} 1 \text{ sum} & : & \frac{4}{5} \text{ sum} \\ 1\frac{1}{2} \text{ comp.} & : & 1 \text{ comp.} \\ \frac{4}{5} \text{ rate} & : & 1 \text{ rate} \\ \frac{4}{5} \text{ time} & : & 1 \text{ time} \end{array}\right\} :: 1 \text{ price} : \frac{24}{31} \text{ of the price.}$

46. $\frac{5}{6}$ of the no., diminished by 22, leaves $\frac{1}{2}$ of the no.; so that 22 is the diff. between $\frac{5}{6}$ of the no. and $\frac{1}{2}$ of it; or $\frac{1}{36}$ of the no. is 22; accordingly, 11 : 36 :: 22 : 72.

47. C contains $\frac{4}{5}$ of D; E = $\frac{3}{4}$ of C = $\frac{3}{20}$ of D; therefore, $\frac{3}{20}$ of D is $1\frac{1}{2}$ gall. less than D; or $\frac{17}{20}$ of D = $1\frac{1}{2}$ gall.; or 17 times D = 30 gall.; &c.

48. A can do $\frac{1}{16}$ per day, or $\frac{1}{32}$ in $\frac{1}{2}$ a day; B can do $\frac{1}{18}$ per day, or $\frac{1}{24}$ in $\frac{3}{4}$ of a day; C can do $\frac{1}{21}$ per day.

A's actual work per day is $\frac{1}{32} + \frac{1}{2}$ of $\frac{1}{32} = \frac{3}{64}$;
B's . . $\frac{1}{18} + \frac{3}{4}$ of $\frac{1}{24} = \frac{22}{360}$;
C's . . . . = $\frac{1}{21}$.

Sum, 945 + 1288 + 960, or 3193...20160ths per day;
3193 parts : 20160 parts :: 1 da. : *Ans.*

PER CENTAGES.

Ex. 17. 3760 : 165 :: 100 : 1650 ÷ 376.

18. 6*s.* 3*d.* : 1*s.* 2*d.* :: 100 : 14 × $\frac{4}{5}$.

19. 7*s.* 5*d.* : 1*s.* 2*d.* :: £100 : £1400 ÷ 89.

20. 780 : 100 :: £6. 16*s.* 6*d.* : £273 ÷ 312.

21. 3500 : 100 :: £128$\frac{4}{7}$ × $\frac{20}{21}$: £$\frac{147 \times 20}{40 \times 21}$.

22. 120 : 100 :: 2*s.* 8*d.* : $\frac{5}{6}$ of 2*s.* 8*d.*;
or, 2*s.* 8*d.* is $1\frac{1}{2}$ of prime cost; therefore, 2*s.* 8*d.* ÷ $1\frac{1}{2}$.

23. 100 − 8 : 100 :: 9*s.* 7*d.* : 100*d.* × $1\frac{1}{2}$.;
or, 9*s.* 7*d.* is $\frac{92}{100}$ of the orig. sum; 115*d.* ÷ ·92.

24. 100 − $2\frac{1}{2}$: 100 :: $9\frac{3}{4}d.$: 975*d.* ÷ 97·5;
or, $9\frac{3}{4}d.$ is $\frac{39}{40}$ of the proposed rate; $9\frac{3}{4}d.$ ÷ $\frac{39}{40}$.

25. $\left.\begin{array}{l} 115 : 100 \\ 500 \text{ yds.} : 1 \text{ yd.} \end{array}\right\} :: £138 : £\frac{6}{25}$.

26. 13*s.* 4*d.* : 15*s.* :: 104 : 117;
or, 13*s.* 4*d.* is $\frac{104}{100}$ of the prime cost; therefore, 15*s.* is $\frac{117}{100}$ of do., indicating a gain of 17 p. c.

27. 2*s.* 3*d.* : 2*s.* 6*d.* :: 99 : 110, or 10 p. c. gain.

28. 7*s.* 8*d.* : 8*s.* 4*d.* :: 115 : 125, or 25 p. c. gain.
7*s.* 8*d.* : 6*s.* $8\frac{1}{2}d.$ :: 115 : 100$\frac{5}{7}$, or $\frac{5}{7}$ p. c. gain.

29. 95 : 125 :: 19*s.* : 125*s.* ÷ 5;
or, $\frac{95}{100}$ of the proposed rate is to equal $\frac{125}{100}$ of 19*s.*

30. $\left.\begin{array}{l} 99\frac{3}{8} : 101\frac{1}{4} \\ 9 \text{ qrs.} : 4 \text{ qrs.} \end{array}\right\} :: 8s.\ 10d. : 4s.$

31. $\frac{1}{4}$ of an Eng. ell is $\frac{5}{16}$ of a yard; therefore, the prime cost was to the selling price as 1 yd. : $1\frac{5}{16}$ yd., or 28 : 33;

33 : 28 : : £4851 : £4116 prime cost:
28 : 33 − 28 : : 100 : $17\frac{6}{7}$ gain per cent.

Simple Interest.

Ex. 25. 4 p. c. per ann. for 3 yrs. = 12 p. c.
£12 : £43. 10*s.* 7$\frac{1}{2}$*d.* : : £100 : £43, &c. × 5 × 20 ÷ 12.

26. $4\frac{1}{2}$ p. c. per ann. for 3 $11\frac{1}{2}$ yrs. = 17·625 p. c.
17·625 : 100 : : £100 : £80000 ÷ 141.

27. £230 : £100 } : : 1 yr. : 200 yrs. ÷ 69.
£3 : £20 }

28. £77$\frac{1}{2}$: £100 } : : 7$\frac{1}{2}$: 90 ÷ 31.
3$\frac{1}{3}$ yrs. : 1 yr. }

29. $2\frac{3}{4}$ p. c. per ann. for 7 yrs. = $19\frac{1}{2}$ p. c.
$119\frac{1}{2}$: 100 : : £149. 1*s.* 3*d.* : £59625 ÷ 477.

30. 6 p. c. per ann. for $\frac{1}{4}$ yr. = $1\frac{1}{2}$ p. c.
101·5 : 100 : : £1042·8 : £208560 ÷ 203.

31. £640. 10*s.* − £618 = £22. 10*s.* interest of both loans.

Int. of £260 for 1 yr. at $3\frac{1}{2}$ p. c. = £ 9. 2*s.*
Do. of £358 for 1 yr. at 5 p. c. = £17. 18*s.*

Int. of both loans for 12 mths. = £27. 0*s.*
£27 : £22$\frac{1}{2}$: : 12 mths. : 10 mths.

Discount for Time.

Ex. 7. $4\frac{1}{2}$ p. c. per ann. for 5 mths. is $1\frac{7}{8}$ p. c., and for 7 mths. it is $2\frac{5}{8}$ p. c.

$101\frac{7}{8}$: 100 : : £86 : £84. 8*s.* 4·122*d.* P.V. of £86;
$102\frac{5}{8}$: 100 : : 95 : 92 11 4·808 P.V. of £95.
£176. 19*s.* 8·93*d.*

8. Int. on £100 for 4 mths. = £$1\frac{1}{3}$; so that he receives £$1\frac{1}{3}$ for lending £$98\frac{1}{3}$ for 4 mths.

$98\frac{1}{3}$: 100 } : : $1\frac{1}{3}$: 300 ÷ 59.
4 mo. : 12 mo. }

9. The present worths of £25, for 3, 6, 9, and 12 months, are as follows:—

$101\frac{1}{2}$: 100 : : £25 : £24. 13*s.* 9·93*d.*
$102\frac{1}{2}$: 100 : : 25 : 24 7 9·66
$103\frac{1}{2}$: 100 : : 25 : 24 1 11·13
105 : 100 : : 25 : 23 16 2·28

P. W. of £100 = £96. 19*s.* 9*d.*

10. $102\frac{1}{2} : 100 :: 30$ gu. $: 29\frac{11}{41}$ gu. P. V. of 30 gu.

$$25 \quad : \quad 4\tfrac{11}{41} :: 100 : \textit{Ans.}$$

Otherwise:—Amt. of 25 guin. in 6 mths. $= 25{\cdot}625$ guin.
$25{\cdot}625 : 30 - 25{\cdot}625 :: 100 : \textit{Ans.}$

11. $101\frac{3}{4} : 100 :: 61s. : 60s.$ P. V. of $61s.$
$102\frac{11}{12} : 100 :: 65s. : 63\frac{3}{13}s.$ P. V. of $65s.$
$60s. : 3\frac{3}{13}s. :: 100 : \textit{Ans.}$

12. $\left.\begin{matrix} 100 : 103\frac{3}{4} \\ 100 : 120 \end{matrix}\right\} :: 15s. \times 100$ yds. $:$ £$4{\cdot}15 \times 22{\cdot}5$

13. The P. V. of the purchase is $\frac{80}{81}$ of £50; the P. V. of the disposal is $\frac{80}{81}$ of £50; and these are as 81 to 83; therefore,

$$81 : 83 - 81 :: 100 : \textit{Ans.}$$

14. $\left.\begin{matrix} 100 : 100\frac{1}{4} \\ 100 : 101\frac{1}{4} \\ 100 : 112\frac{1}{2} \end{matrix}\right\} :: £75 : £\frac{241 \times 81 \times 9}{16 \times 16 \times 8}$

15. The P. V. of the purchase is to that of the disposal as $100 : 101\frac{20}{41}$, or as 71 : 72; but the P. V. of the purchase is $\frac{100}{105}$ or $\frac{20}{21}$ of £23; therefore,
$71 : 72 :: \frac{20}{21} : \frac{480}{497}$ of £23 is the P. V. of the disposal, the disc. being $\frac{17}{497}$; accordingly, $480 : 17 :: 100 : 3\frac{13}{24}$ is the rate p. c. for the time sought; and 5 p. c. : $3\frac{13}{24}$ p. c. :: 12 mo. : $8\frac{1}{2}$ mo.

16. $99 : 100 :: 16s.\ 6d. : 16s.\ 8d.$ the prime cost;
$16s.\ 8d. : 4d. :: 100 : 2$ the rate p. c. for credit;
5 p. c. : 2 p. c. :: 12 mo. : *Ans.*

Division into Proportional Parts.

Ex. 5. $10 + 12\frac{1}{2} : 17, 10,$ and $12\frac{1}{2} :: 15 :$ *Answers.*

6. The payments of A, B, C, and D, are as 1, 2, 3, and 5;
$11 : 1, 2, 3,$ and $5 ::$ £2607 : *Answers.*

7. $15 + 17 : 15$ and $17 ::$ £14. 8s. : *Answers.*

8. James's share is to mine as $11 : 3$; therefore,
$11 + 3 : 11$ and $3 :: 2s.\ 4d. :$ *Answers.*

9. The shares of A, B, and C, are as 2, 5, and $\frac{3}{4}$;
$2\frac{3}{4} : 2, 5,$ and $\frac{3}{4} :: 5\frac{1}{2}d. :$ *Answers.*

10. $63 - 31 : 63 + 31 :: 4$ lbs. : $11\frac{3}{4}$ lbs.

11. One boy gets 1 quantity,
One woman 3 such quantities,
Each of 2 men gets 6, $= 12$ ditto.

The whole weight $= 16$ ditto.
$16 : 1, 3,$ and $6 ::$ 272 lbs. : *Answers.*

12. **The Nos.** are as 17 : 7; therefore,
24 : 17 and 7 :: 42 : *Answers.*

13. The greater will be to the less as 39 to 1;
40 : 39 and 1 :: 39 : *Answers.*

14. The No. of women is $\frac{1}{8}$ of the men; the boys $\frac{3}{8}$ of $\frac{1}{8}$ of ditto; the girls $\frac{5}{10}$ of $\frac{3}{8}$ of $\frac{1}{8}$ of ditto. Therefore, the men, women, boys, and girls, are in number as 1, $\frac{1}{8}$, $\frac{1}{26}$, and $\frac{1}{120}$, or as 360, 45, 10, and 3;
418 : 360, 45, 10, and 3 :: 2090 : *Answers.*

15. The one is to the other as $\frac{3}{4}$: $\frac{2}{3}$ or 9 : 8;
17 : 9 and 8 :: 85 : *Answers.*

16. The first is $\frac{3}{10}$ of the fourth; the third is $\frac{3}{10}$ of $\frac{3}{10}$ of ditto; the second is $1\frac{3}{10}$ of $\frac{3}{10}$ of ditto; therefore the 1st, 2nd, &c., are as ·3, ·39, ·09, and 1, or as 30, 39, 9, and 100;
178 : 30, 39, 9, and 100 :: 66$\frac{2}{3}$*d.* : *Answers.*

17. B pays $\frac{11}{12}$ of C; A $\frac{7}{8}$ of $\frac{11}{12}$ of ditto; D $\frac{9}{10}$ of $\frac{7}{8}$ of $\frac{11}{12}$ of ditto; hence, A, B, C, D, pay in proportion as $\frac{77}{96}$, $\frac{11}{12}$, 1, and $\frac{231}{336}$, or as 770, 880, 960, and 693;
3303 : 770, 880, &c., :: 45*s.* 10$\frac{1}{2}$*d.* : *Answers.*

18. A : B :: 15 : 12, or 5 : 4; B : C :: 10 : 15, or 4 : 6; 5 + 4 + 6 : 5, 4, and 6 :: 60*s.* : *Answers.*

19. 1 boy gets 1 share; or 11 boys get 11 shares,
1 woman 3 ditto; or 7 women 21 ditto,
1 man 12 ditto; or 4 men 48 ditto,
80 sh. : 1, 3, and 12 sh. :: £79. 18*s.* 4*d.* : *Answers.*

20. B $\frac{4}{5}$ of A per hour; C $\frac{5}{8}$ of ditto; D $\frac{11}{16}$ of $\frac{4}{5}$ of ditto; therefore the proportions are 1, $\frac{4}{5}$, $\frac{5}{8}$, and $\frac{22}{25}$, or 200, 160, 225, and 176; consequently, 225 — 176 : 761 :: 14 pa. : *Ans.*

Fellowship.

Ex. 4. 50 : 8 and 5 :: £175 : *Answers.*

5. D shares $\frac{1}{4}$ of £154, E $\frac{1}{7}$ of it, F $\frac{3}{11}$ of it; and G shares the remainder = £154 — £80. 16*s.*

6. £1074 — 150, or £924 divided among A, B, and C, in proportion to their stocks, gives A £528, which added to his salary makes his receipt £678; &c.

7. The shares of loss are as 530 × 10, 480 × 12, and 556 × 15; or as 530, 576, and 834;
1940 : 530, &c. :: £271·6 : *Answers.*

8. 27 oxen for 75 da. = 75 times 27 oxen for 1 day;
56 oxen for 39 da. = 39 times 56 oxen for 1 day;
or the proportions are 75 × 9 and 13 × 56;
1403 : 675 and 728 :: £61 : *Answers.*

9. The proportions are 620 × 14, 490 × 15, and 420 × 10; or 62 × 2, 7 × 15, and 6 × 10;
289 : 124, 105, and 60 :: £170 : *Answers.*

10. The greater share is $\frac{5}{14}$ of the joint capital; hence, the less share must be $\frac{4}{14}$; or the stocks are as 9 : 5;
9 — 5 : 9 and 5 :: £100 : *Answers.*

11. $1\frac{1}{2}$ ox. : 7 ox. :: 16 sh. : $74\frac{2}{3}$ sh. = 7 ox.;
also, 6 horses = 96 sheep; so that B's proportion is $74\frac{2}{3} \times 6 + 96 \times 4 = 832$, and A's $120 \times 5 = 600$;
1432 : 600 and 832 :: £35 : *Answers.*

12. A gains £10 a month, B £$10\frac{2}{3}$ a month; therefore A's stock is to B's as 12 : 13;
12 + 13 : 12 and 13 :: £1000 : *Answers.*

13. A gains £90 ÷ 11 per month, B £104 ÷ 15 per month; therefore, A's stock is to B's as 45 ÷ 11 to 52 ÷ 15, or as 675 : 572;
675 — 572 : 675 and 572 :: £103 : *Answers.*

14. A's gain is $\frac{13}{60}$ of his stock, B's $\frac{17}{60}$, C's $\frac{12}{60}$; consequently, the times are as 13, 17, and 12;
17 — 12 : 13, 17, and 12 :: 5 mo. : *Answers.*

15. $1\frac{1}{2}$ p. c. to C, deducted from £5680, makes the gain £1094. 16*s.* to be divided according to the stocks, viz. as 4, 3, and 2.
9 : 4 :: £1094. 16*s.* : £486. 11*s.* $6\frac{2}{3}$*d.* A's gain;

A resumes the stock,	4500	0	0
$4\frac{1}{2}$ p. c. on £2500,	112	10	0
A receives	£5099	1	$6\frac{2}{3}$

9 : 3 :: £1094. 16*s.* : £364 18 8 B's gain,

$4\frac{1}{4}$ p. c. on £1500,	67	10	0
B receives	£297	8	8

C gets the remaining sum, viz. £283. 9*s.* $9\frac{1}{3}$*d.*

Miscellaneous Questions.

Ex. 1. (56 + 59 + 61)*s.* ÷ 3 = 58*s.* 8*d.*

2. S. + P. = $1\frac{1}{2}$ S.; therefore, 216000 ÷ 1·2 = S.

3. 5 qrs. : 4 qrs. :: 3*s.* $5\frac{1}{2}$*d.* : 2*s.* 9*d.* a yard;
175*s.* ÷ (3*s.* 6*d.* + 2*s.* 9*d.*) = 28 yds.

4. The chain cost 1 sum; the watch cost 1 such sum + £11. 10*s.*; therefore 2 such sums + £11. 10*s.* amount to £20, or £8. 10*s.* ÷ 2 = £4. 5*s.* cost of the chain.

Otherwise:—The watch cost 1 sum; the chain cost 1 such sum

— £11. 10*s.*; therefore 2 such sums — £11. 10*s.* amount to £20, or £31. 10*s.* ÷ 2 = £15. 15*s.* cost of the watch.

5. $(37s. + 59s.\ 6d. + 100s.) \div (2 + 3 + 5) = Ans.$

6. The greater is to the less as $13\frac{1}{2}$ to 1, or 27 to 2;
$27 - 2 : 27$ and $2 :: 13 \cdot 5 : Answers.$

7. An acre is 4840 sq. yds.; $\sqrt{4840} = Ans.$

8. The prime cost must be $\frac{5}{8}$ of the selling price;
$5 : 8 :: 4s.\ 7d. : 7s.\ 4d.$

9. Half the circuit, or 52 ft., is the sum of length and breadth; 52 + 18 makes twice the length, or 52 — 18 makes twice the breadth.

10. 528 yds. = ·3 mile; therefore G travels ·3 × 5 = $1\frac{1}{2}$ mile more than S, and as both together travel $36\frac{1}{2}$ mi., G's journey is $(36\frac{1}{2} + 1\frac{1}{2}) \div 2$, or 19 mi., and his rate is 19 ÷ 5 = 3·8 mi. per hour; also S's rate is ·3 mile less = 3·5 mi. per hour.

11. 7*s.* a lb. is $10\frac{1}{2}d.$ for 2 ounces;
100 : 110 :: $10\frac{1}{2}d.$: 11·55*d.* proposed selling price of 2 oz., from which deduct 1 oz. at 6*d.*, and there remains for the price of the other ounce 5·55*d.*

12. $\frac{40 \cdot 75 \times 223}{20 \times 365} = £363 \cdot 49 \div 292 = 1st\ Ans.$
$57 \cdot 6375 \times 2 \cdot 2$ yrs. ÷ 20 = £57·6375 × ·11 = 2nd *Ans.*

13. The *constant* diff. of the ages is 40 — 9 or 31;
3 — 1 : 3 :: 31 : $46\frac{1}{2}$ the father's anticipated age, which is $6\frac{1}{2}$ yrs. hence.

14. $\sqrt{65792 \cdot 25} = 256 \cdot 5$ yds. at 256·5*d.*

15. $103\frac{1}{2} : 100 :: £150 : £\frac{10000}{69}$ B paid;
$104\frac{3}{4} : 100 :: £\frac{10000}{69} : \frac{1000000}{23 \times 314} = Ans.$

16. £46 × 1·0525 = £48·415, the amount; of which B gets 1 share, A 1 share — 3*s.* 6*d.*, and C 4 shares; in all, 6 shares — £·175 = £48·415.
£48·59 ÷ 6 = £8. 1*s.* 11·6*d.* to B; &c.

17. (26 + 2) ÷ 2 = 14 mi. the greater distance; 12 mi. the less; divide 36 in. into two parts as 14 : 12, or 7 : 6.

18. The second No. is 2 times the first and 1 more; the third No. is 3 times the second and 3 more, or 6 times the first and 6 more; therefore the sum of the three Nos. is 9 times the first and 7 more; accordingly, (70 — 7) ÷ 9 gives the first No.

19. 107 — 83 : 100 :: 5*s.* : 20*s.* 10*d.*

20. The No. in the highest class is contained in each of the other classes, with 2 boys additional in the second class, 4 in the third, 6 in the fourth, 8 in the fifth. Therefore 100 boys = 5 times

the No. in the first class and 20 boys more; or 80 boys = 5 times the No. of the first class.

21. 6 horses and 7 cows = 2 horses more and 3 cows fewer; so that the worth of 2 horses is also the worth of 3 cows; and hence, 4 horses = 6 cows; accordingly, 6 + 10 or 16 cows sold for £165. 6*s.* 8*d.*, which is £10. 6*s.* 8*d.* for each cow.

22. $\sqrt[3]{(277\frac{1}{2} \times 160)} = \sqrt[3]{44360}$.

23. 97 : 100 :: 400 lbs. : (40000 ÷ 97) lbs. must be fused in proportions of 41, 55, and 24;

120 : 41, 55, and 24 :: 40000 ÷ 97 : *Answers.*

24. The floor of the room required will be divisible into two squares whose sum is the given area; therefore, $52\frac{1}{2} \times 14\frac{1}{7}$ ft. will be the area of each square; and $\sqrt{752 \cdot 5}$ will be the required width.

Also, $(105 \times 28\frac{3}{4}) \div (22\frac{1}{2} \times \frac{2}{3}) = \frac{23}{12}$ of 344, which will give the No. of planks.

25. 15 : 4, 6, and 5 :: 75 bu. : *Answers* in bushels.

20 bu. oats	at 1 price	= 20 prices,
30	barley at $1\frac{1}{4}$ ditto	= $37\frac{1}{2}$ ditto,
25	wheat at $2\frac{1}{2}$ ditto	= $62\frac{1}{2}$ ditto,
Therefore £18 is		120 prices of oats.

360*s.* ÷ 120 = 3*s.* per bu. for the oats.

26. B's gain is $\frac{5}{8}$ of C's; A's gain $\frac{4}{7}$ of $\frac{5}{8}$ of C's; or the gains of A, B, and C, are as $\frac{10}{21}$, $\frac{5}{8}$, and 1, or as 20, 35, and 42; consequently, the stocks are as 20 ÷ 7, 35 ÷ 9, and 42 ÷ 10, or as 900, 1225, and 1323.

3448 : 900, &c. :: £1000 : *Answers.*

27. For 10 dozen I receive 10 times the selling price per dozen; and if £100 gain 10 times the selling price, 8*s.* must gain $\frac{1}{25}$ of the selling price, or the prime cost is $\frac{24}{25}$ of the selling price.

24 : 25 :: 8*s.* : 8*s.* 4*d.* per dozen.

28. Putting 2 as the proportional value of the first, 5 will represent the fourth, 2·8 the second, 3·92 the third;

6·744 : 2, 5, &c. :: 281 : *Answers.*

29. A $\frac{1}{10}$ + B $\frac{1}{6}$ + C $\frac{1}{12}$ = $\frac{53}{160}$ of the work per day;

$\frac{1}{10} \times 3$ da. + $\frac{1}{6}$ = $\frac{107}{160}$ done before C begins.

Remainder $\frac{53}{160}$ to be done by the three conjointly, at the rate of $\frac{53}{160}$ per day; this will require 2 days, which added to the 3 days preceding C's commencement gives 5 days altogether.

*** *For solutions of the remaining Misc. Questions in the Manual,* see *Misc. Examples,* p. 200 et seq.

APPENDIX V.

Remarks on the Construction of some of the Exercises in the Manual, under the Simple Rules of Addition, Multiplication, and Division.

In the Manual, one-half of the Exercises in Simple Addition, Multiplication, and Division, are so constructed as to admit the application of an *Artificial Test*, enabling the Teacher to ascertain, with great ease and expedition, the accuracy of the results presented by the Pupils. This may be found, occasionally, a useful accommodation; because for such Questions there will be no need to refer to the Manual for the Answers.

The 2nd, 4th, 6th, &c., Exercises, viz. all those of which the numbering is even, are constructed for the application of the Test.

To test the Results in Addition.

Read, in succession from the top, the figures in units column, and if you reckon each figure 1 less as you proceed, you will have read the figures of the Answer.

Thus, in the 38th Exercise, four figures taken from the top of units column give 7369, which, being read each 1 less, give 6258, the Answer.—Observe, that (in the Test Exercises in Addition) 1 less than 0 is reckoned 9, without carrying; accordingly, in the 44th Exercise, the testing figures 59060 are read 48959.

To test the Results in Multiplication.

From each figure in half the number of figures, on the left of the product, deduct 1; the remainders will be the other half of the product figures.

Thus, in the 66th Exercise, half the figures on the left of the product are 52176, which being read as 1 less, we have 41065, the figures which constitute the remaining half of the product.

To test the Results in Division.

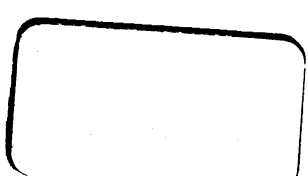
Let the figures of the integral part of the quotient, along with those of the remainder, or numerator of the fractional part (if any), be divided into two equal periods, and tested as in Multiplication.

Thus, in the 44th Exercise, the quotient, with remainder annexed, is 48537,4; where we have 485 − 111 = 374.

The Teacher may, if he think it expedient, easily construct sets of other Exercises in Addition and Division, conformable to the artificial test, and have them written, without answers, on sheets available for use in a class. He will find, however, that *experimental* construction is impracticable for Multiplication; and therefore, we shall here supply a few additional Exercises, derived from a mathematical investigation which determines their conformity to the test.*

1.	92342	×	784	8.	5975108	×	806
2.	80647	×	593	9.	3720786	×	920
3.	39620	×	654	10.	3575723	×	675
4.	312357	×	279	11.	364743	×	4589
5.	167385	×	369	12.	459763	×	8407
6.	135702	×	485	13.	766438	×	6253
7.	8590140	×	534	14.	362432	×	7136

* For information respecting the use of artificial tests, see the Author's 'Exercises in the first Four Rules of Arithmetic.'

THE END.

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